

## CROSS-LAYER MODELING AND OPTIMIZATION OF MULTI-CHANNEL COGNITIVE RADIO NETWORKS UNDER IMPERFECT CHANNEL SENSING

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**ABSTRACT.** In this paper, we consider a multi-channel cognitive radio network with multiple secondary users (SUs) and analyze the performance of users in the network. We assume primary users (PUs) adopt the automatic repeat request (ARQ) protocol at the medium access control layer. We have two main goals. Our first goal is to develop a cross-layer performance model of the cognitive radio network by considering the retransmission characteristics of the ARQ protocol and the interference between PUs and SUs due to imperfect channel sensing. Using the cross-layer performance model we analyze the throughput performance of SUs and the delay performance of PUs.

Our second goal is to propose an optimal channel sensing method that maximizes the throughput performance of SUs while a given delay requirement of PUs is guaranteed. To this end, using our cross-layer performance model, we formulate an optimization problem and solve it to get an optimal channel sensing method that satisfies the design objectives. Numerical and simulation results are provided to validate our analysis and to investigate the performance of the optimal channel sensing method.

**1. Introduction.** As the number of new wireless services has been rapidly increased, available radio spectrum resources have become scarce. In addition, recent measurements of spectrum usage by the FCC show that most spectrum resources have been underutilized [7, 8]. The concept of cognitive radio (CR) has been proposed to solve this scarcity and underutilization problem in spectrum resources [18]. The basic concept of CR networks is as follows: A new network is allowed on top of a preexisting network. Even though the users in the new network, called secondary users (SUs), have no license for the spectrum resources of the preexisting network, they are allowed to utilize unused spectrum resources as long as they cause no interference to the licensed users, called primary users (PUs). In the last decade, a number of research works on CR networks have been published in the

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open literature. The IEEE 802.22 standard aiming at how to utilize CR techniques is completed recently, so that CR networks are expected to be used to accommodate more wireless applications in the network.

In this paper we consider a multi-channel CR network with multiple SUs. We consider a generic channel sensing method for SUs where each SU uses a predetermined threshold to determine whether a channel is busy (i.e., occupied by a PU) or not. Each SU determines the state of the channel by comparing the threshold with a detection metric obtained by measuring the channel. Obviously, there exists the possibility of errors in channel sensing, i.e., the misdetection and false alarm probabilities, and the imperfect channel sensing significantly affects the performance of SUs as well as that of PUs.

We have two main goals. Our first goal is to develop and analyze a cross-layer performance model for the CR network where PUs adopt the automatic repeat request (ARQ) protocol at the medium access control (MAC) layer and the channel sensing of SUs is imperfect. In most of the existing studies on CR networks, the channel occupancy process by PUs are assumed to be a *predetermined* stochastic process, for instance, Bernoulli process or two state Markov process [9, 13, 23, 28]. However, such assumption does not capture the direct interference between PUs and SUs. Moreover, most existing research works do not consider the characteristics of the protocols implemented at PUs. For instance, suppose that PUs use the ARQ protocol which is widely used in wireless networks. In this case, when there occurs interference in PUs' transmission by SUs due to imperfect channel sensing of SUs, the interfered information (packet) of PUs is retransmitted by the ARQ protocol and consequently the channel occupancy process by PUs is affected by such retransmissions. Hence, the channel occupancy process of a PU cannot be *a priori* given. Furthermore, the more retransmissions occur, the more busy channels SUs have, which consequently affects the performance of SUs. With the above observation, we develop a cross-layer performance model where the queueing performance of a PU is considered to model the direct interaction between PUs and SUs due to imperfect channel sensing of SUs and the characteristics of the ARQ protocol of PUs. Using our cross-layer performance model, we can obtain the throughput performance of SUs and the delay performance of PUs.

Based on the cross-layer performance model, our second goal is to propose an optimal channel sensing method that maximizes the throughput performance of SUs, while guaranteeing the requirement for PUs on packet delay. That is, we will determine the optimal threshold value in the channel sensing of SUs with which each SU achieves its maximum throughput performance while a given delay requirement of PUs is guaranteed. We use our cross-layer performance model to investigate the relation between throughput performance and the threshold and then find the optimal threshold value from the cross-layer viewpoint. Note that this is a cross-layer design problem because we propose an optimal channel sensing method at the physical (PHY) layer by considering the PHY layer performance (the channel sensing errors) as well as the MAC layer performance (throughput and delay) simultaneously in the problem.

From our cross-layer analysis we show that the use of a strict requirement for channel sensing, for instance the sensing requirement in the IEEE 802.22 standard, is not suitable for distributed CR networks and the optimal channel sensing policy should be designed from the cross-layer viewpoint, which is the main contribution of this paper. For details, refer to Section 6.

There are a number of previous works to improve the performance of SUs. Within the scope of the MAC layer, most research works in the literature focus on medium access protocols, e.g., [4, 10]. For the PHY layer performance, there are also a number of research works, most of which are devoted to design efficient and accurate algorithms for detecting PU signals, e.g., [26, 27]. We will provide a brief summary on existing works in Section 2.

The remainder of this paper is organized as follows: In section 2, we present a brief summary on related works. In section 3, we provide a description of our network model. In section 4, we develop a cross-layer performance model to consider the interaction between PUs and SUs. In section 5, we analyze the performance of users based on our cross-layer performance model. In section 6, we consider and analyze a cross-layer optimization problem to get an optimal channel sensing method. We also investigate the performance of the optimal channel sensing method through numerical and simulation studies in the section. In Section 7, we investigate the impact of the interference from SUs on the cross-layer performance model. Finally, we give our conclusions in section 8.

**2. Related Works.** Channel sensing at the PHY layer is one of the key functionalities in CR networks and sensing accuracy is an important performance metric at the PHY layer. A number of channel sensing techniques are proposed in the literature, e.g., the energy-based detector, the cyclostationary feature detector, the matched filter [24, 27]. In addition, in order to improve sensing accuracy cooperative sensing techniques are proposed [2, 26], and Singh et. al. find the threshold of a cooperative sensing method that minimizes the average of false alarm probability and misdetection probability under single channel CR networks with two SUs [20]. Although sensing accuracy is important in CR networks, there exists a tradeoff between sensing time and performance. In [15], Liang et al. provide the optimal design of sensing time subject to the constraint on the false alarm and misdetection probabilities. In [13], a framework for channel sensing is provided to maximize spectrum efficiency by using the optimal sensing and transmission durations. In [19], Peh et al. consider a single channel CR network with a fusion center and propose an optimization algorithm that controls the parameters of a cooperative sensing to maximize the throughput of an SU.

From the viewpoint of MAC layer performance in CR networks, it is important to distribute the channel accesses of SUs to alleviate packet collision between SUs as well as interference to PUs. There are many works on the design of CR MAC protocols [16, 17, 21, 28]. Also, there are several works about distributed random access policies in which SUs determine their channel access probabilities. In [23], Wang et. al. consider time slotted CR networks with multiple channels and multiple SUs. The delay performance is analyzed based on fluid flow approximation and Poisson driven stochastic differential equations. In [9], the queueing performance of SUs in multi-channel CR networks is analyzed by using the effective bandwidth approach. In order to improve the performance, they suggest a random access policy that stochastically determines channel access probabilities based on the number of idle channels. Both studies propose the optimal access probabilities that maximize the performance of SUs. However, channel sensing errors at the PHY layer are not considered in [23] and [9]. In CR networks, channel sensing errors are not avoidable and affect performances of users. In [6], they investigate the effect of channel sensing errors on the stability region of users for a single channel CR networks with multiple

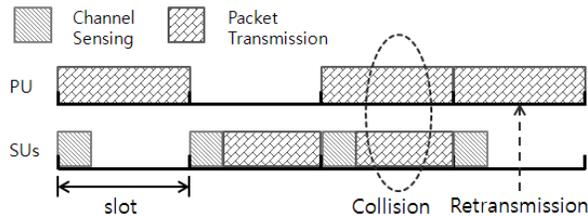


FIGURE 1. Channel sensing and packet transmission.

PUs and SUs. They show that the stability region is significantly degraded due to channel sensing errors. To alleviate the effect of channel sensing errors, they propose an adaptive channel access policy in which channel access probabilities depend on the received signal power.

**3. System Description.** We consider a time slotted system with  $N$  primary users (PUs) and  $M$  secondary users (SUs). There are  $N$  channels such that each channel is licensed to each PU. The time domain is slotted with equal duration and time slots of all users are synchronized.

At each slot, a user who has a packet in the queue at the MAC layer transmits the packet via a channel. The packet may not be successfully transmitted to its receiver due to interference from other users. In other words, when there are two or more users transmitting their packets through the same channel simultaneously, all transmitted packets are collided. We want to investigate the impact of the interaction between PUs and SUs from throughput and delay performance viewpoint. We therefore assume that the packet transmission fails only when there occurs a collision. That is, there are no other sources of packet loss such as link failure, noise or channel fading during the packet transmission.

For a reliable packet transmission, all PUs use the ARQ protocol. Thus, if a packet transmission of a PU fails due to the interference by SUs, the PU retransmits the interfered packet until it is successfully transmitted. For simplicity, we assume that a PU knows whether its packet transmission is successful or not at the end of the slot where the transmission occurs. So the PU can retransmit immediately the interfered packet at the next slot.

Each SU independently tries to exploit channels. At the beginning of each slot, an SU randomly selects a channel and performs the channel sensing to determine whether the channel is idle or not. After the channel sensing, the SU transmits its packet if the channel is sensed as idle. However, due to the limitation in channel sensing, the channel sensing result potentially contains an error. There are two types of errors. The first type is the false alarm, which implies that the SU senses an idle channel as busy. The second type is the misdetection, which implies that the SU senses a busy channel as idle. While the false alarm makes the SU lose its opportunity for transmission, the misdetection causes a packet collision between the SU and a PU. In addition, the misdetection makes the channel busier due to the packet retransmission by the PU. Both errors might significantly affect the performance of PUs and SUs. Thus, it is important to analyze the impact of channel sensing errors on the performance of PUs and SUs. To this end, we assume that all SUs always have packets to transmit, i.e., all SUs are saturated from now on.

In the following, the PU network and the SU network are assumed to be homogeneous. In other words, users in the same network have the same system parameters, for example, all PUs have the same packet arrival rate and all SUs have the same misdetection probability and the same false alarm probability.

#### 4. Mathematical Modeling.

**4.1. Channel Sensing.** In order to exploit channel resources, it is important to correctly sense whether a channel is idle or not with high probability. A number of channel sensing methods have been proposed in the literature to improve the accuracy of channel sensing. This paper considers a generic channel sensing method, so that our mathematical model is independent of the details of channel sensing methods. Most existing channel sensing methods decide the channel state by comparing a predetermined threshold  $\tau$  with a decision metric  $D$  which is obtained by measuring the channel [26, 29]. Without loss of generality, we assume that the generic channel sensing method decides the channel is idle if  $D < \tau$  and the channel is busy if  $D \geq \tau$ .

A channel sensing method can be mathematically formulated as a binary hypothesis test. Let  $\mathcal{H}_0$  and  $\mathcal{H}_1$  refer to the hypothesis on the idle channel state and the hypothesis on the busy channel state, respectively. Then the false alarm and misdetection probabilities of the generic channel sensing method are given by

$$p_f := Pr\{D > \tau | \mathcal{H}_0\}, \quad (1)$$

$$p_m := Pr\{D < \tau | \mathcal{H}_1\}. \quad (2)$$

In order to construct a rigorous analytic model, we assume that  $p_f(\tau)$  and  $p_m(\tau)$  are continuous in  $\tau$  and have their inverse functions. To the best of our knowledge, these two assumptions hold for most existing channel sensing methods. From (1) and (2) we know that  $p_f = p_f(\tau)$  is a decreasing function in  $\tau$  and  $p_m = p_m(\tau)$  is an increasing function in  $\tau$ . This implies that, if we increase  $p_f$  (by decreasing  $\tau$ ), then  $p_m$  decreases. Accordingly, there is a continuous decreasing function  $f : [0, 1] \rightarrow [0, 1]$  such that  $p_m = f(p_f)$ , which will be used later.

#### 4.2. Channel Occupancy Modeling based on Imperfect Channel Sensing.

In this subsection, we develop a mathematical model to describe the channel occupancy of PUs by using a Markov chain. Since we assume that each PU has a license to access its designated channel, each PU transmits its packet through the channel whenever it has a packet to transmit. This implies that the channel occupancy process of a PU is completely determined by the queueing process of the PU. So we consider the queueing process of a PU to model the channel occupancy process. Each PU has a queue at the MAC layer in order to store its packets. We tag an arbitrary PU as our reference and we consider the queueing process of the tagged PU. Since the consideration of a CR network implicitly implies that each PU in the CR network has a small packet arrival rate, the queue size is less important in the analysis of CR networks. Consequently, we assume the queue size is infinite in this paper.

Each PU generates a packet at the beginning of a slot. Let  $A_t$  ( $t = 1, 2, \dots$ ) denote the number of generated packets at the beginning of slot  $t$ . The packet arrival process  $\{A_t, t \geq 1\}$  is modulated by a two state stationary Markov chain

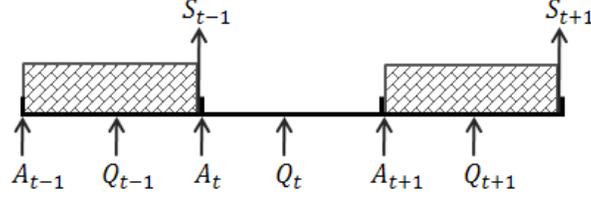


FIGURE 2. Illustration of the queuing process of the tagged PU.

with state space  $\{0, 1\}$  and transition probability matrix

$$R := \begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}.$$

When the Markov chain of  $A_t$  is in state 0 (state 1, resp.) at slot  $t$ ,  $A_t = 0$  ( $A_t = 1$ , resp.).

The stationary probability vector  $\phi$  of  $R$  (i.e.  $\phi R = \phi$ ) is given by

$$\phi = (1 - \lambda, \lambda), \quad \lambda = \frac{a}{a+b}.$$

Since one packet is generated at a slot with state 1, the average packet arrival rate of the tagged PU is equal to  $\lambda$ .

Let  $Q_t$  ( $t = 1, 2, \dots$ ) denote the number of packets in the queue of the tagged PU at the middle of slot  $t$ . Let  $S_t$  ( $t = 1, 2, \dots$ ) denote the *service capacity of the tagged PU* to transmit packets during slot  $t$ . Thus,  $S_t = 1$  if no SU transmits via the designated channel of the tagged PU, called the tagged channel, during slot  $t$ , and  $S_t = 0$  otherwise. Since a packet transmission of the tagged PU is completed at the end of a slot, the packet is removed from the queue at the end of the slot if the packet is successfully transmitted at the slot. The queuing process is illustrated in Fig. 2. Then, the queuing process  $\{Q_t, t \geq 1\}$  satisfies the following equation:

$$Q_1 = 0, \quad Q_t = (Q_{t-1} - S_{t-1})^+ + A_t, \text{ for } t \geq 2, \quad (3)$$

where  $a^+$  is defined by the maximum value of  $a$  and 0.

The success of a packet transmission of the tagged PU depends on the decisions of SUs who select the tagged channel at slot  $t$ .  $S_t$  depends on the number of SUs that select the tagged channel at slot  $t$ , denoted by  $M_t$ . Since each SU randomly selects a channel,  $\{M_t, t \geq 1\}$  are independent and identically distributed (i.i.d.) with binomial distribution having parameters  $M$  and  $1/N$ .

The channel access by an SU depends on its channel sensing result. When the tagged channel is busy at slot  $t$  (i.e.  $Q_t > 0$ ), an SU transmits with probability  $p_m$

due to its misdetection of the busy channel. It immediately follows that

$$\begin{aligned}
Pr\{S_t = 1 | Q_t > 0\} &= \sum_{k=0}^M Pr\{S_t = 1 | M_t = k, Q_t > 0\} Pr\{M_t = k | Q_t > 0\} \\
&= \sum_{k=0}^M (1 - p_m)^k \binom{M}{k} \left(\frac{1}{N}\right)^k \left(\frac{N-1}{N}\right)^{M-k} \\
&= \left(\frac{N-1}{N} + \frac{1-p_m}{N}\right)^M \\
&= \left(1 - \frac{p_m}{N}\right)^M.
\end{aligned} \tag{4}$$

The second equality holds because  $Q_t$  and  $M_t$  are independent. Note here that  $Pr\{S_t = 1 | Q_t > 0\}$  decreases in  $p_m$ , which will be used later.

We now denote  $p'_m$  by the probability that at least one of SUs transmits through the tagged channel when the channel is busy. By its definition, we have

$$\begin{aligned}
p'_m &= 1 - Pr\{S_t = 1 | Q_t > 0\} \\
&= 1 - \left(1 - \frac{p_m}{N}\right)^M.
\end{aligned} \tag{5}$$

So each packet of the tagged PU is successfully transmitted with probability  $1 - p'_m$  and is interfered by SUs with probability  $p'_m$ . This implies that the number of slots needed to successfully transmit a packet, called the transmission time of a packet, follows the geometric distribution with parameter  $p'_m$ . Therefore, the offered load of the queueing process  $\{Q_t, t \geq 1\}$ , which is defined by the fraction of its arrival rate over its service rate, is given by  $\lambda/(1 - p'_m)$ . Since the transmission times of packets are i.i.d., the steady state probability  $\pi_{busy}$  that the tagged channel is busy at an arbitrary slot is equal to the offered load, provided that the queueing process  $\{Q_t, t \geq 1\}$  is stable, i.e.,  $\lambda < 1 - p'_m$ . The stable condition of  $\{Q_t, t \geq 1\}$  will be discussed in section 5.1. We thus have

$$\begin{aligned}
\pi_{busy} &= \frac{\lambda}{1 - p'_m}, \\
\pi_{idle} &= 1 - \frac{\lambda}{1 - p'_m},
\end{aligned} \tag{6}$$

where  $\pi_{idle}$  is the probability that the tagged channel is idle at an arbitrary slot. From (5), we know  $p'_m$  is monotonically increasing in  $p_m$ . Consequently,  $\pi_{idle}$  is monotonically decreasing in  $p_m$ .

Note that the transmission time has no limit in our model, while the ARQ protocol in practice always has an upper limit for the number of retransmissions. The throughput of an SU and the delay of a PU without an upper limit become worse than those with an upper limit. So our model provides lower bounds of the performance metrics of interest for the ARQ protocol. However, this lower bounds are tight because the transmission time of a packet follows the geometric distribution with parameter  $1 - p'_m$ . For instance, let  $K$  be the upper limit of the ARQ protocol. Then the probability that a packet continuously fails to be transmitted  $K$  times, is  $p_m'^K$  which decays exponentially fast. That is, most of the packets are transmitted successfully before the retransmission time reaches the upper limit  $K$  even when  $K$  is not so large. In fact, we confirm that the impact of the upper limit is negligible in

our model through simulation even though we do not include our simulation results in this paper to save space. Since the consideration of the upper limit makes the analysis more complex without any significant merit, we do not consider the upper limit for the retransmission in our model.

Before we end this section, it is worth mentioning the following remark. In many existing studies in the literature, e.g. [9, 13, 14, 23, 28], they assume that the channel occupancy process by PUs are according to a *predetermined* stochastic process, e.g., a 2-state Markov chain, and do not consider the interference from SUs due to imperfect channel sensing. However, the probabilities related to the channel occupancy process,  $\pi_{busy}$  and  $\pi_{idle}$ , given in (6) are obtained in our model after considering the interference from SUs due to imperfect channel sensing. This shows our model is more practical than those in the existing studies. Later in Section 7, we will show that the consideration of interference from SUs in the channel occupancy model is important through numerical analysis and simulation.

**5. Performance Analysis.** In this section, we first analyze the delay performance of a PU, and then analyze the throughput performance of an SU.

**5.1. Delay Analysis for a PU.** The packet delay of the tagged PU is defined by the time duration from the arrival epoch of an arbitrary packet of the tagged PU to the successful transmission epoch of the packet. To obtain the packet delay, we now revisit the queueing process  $\{Q_t, t \geq 1\}$  given in (3) and analyze the queueing behavior of the tagged PU. Note that the queueing process  $\{Q_t, t \geq 1\}$  in (3) itself is unfortunately not a Markov chain. So we introduce an auxiliary process  $\{A_t, t \geq 1\}$  to form the stochastic process  $\{(Q_t, A_t), t \geq 1\}$ , which is a Markov chain with state space  $\mathcal{S} = \{0, 1, 2, \dots\} \times \{0, 1\}$  and transition probability matrix  $P = (P_{i,j}, i, j \in \mathcal{S})$  of the form

$$P := \begin{pmatrix} U_1 & U_0 & O & O & O & \dots \\ V_2 & V_1 & V_0 & O & O & \dots \\ O & V_2 & V_1 & V_0 & O & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

where each component is a  $2 \times 2$  matrix and

$$\begin{aligned} U_1 &:= \begin{pmatrix} 1-a & 0 \\ b & 0 \end{pmatrix}, \\ U_0 &:= \begin{pmatrix} 0 & a \\ 0 & 1-b \end{pmatrix}, \\ V_2 &:= (1-p'_m)U_1, \\ V_1 &:= p'_m U_1 + (1-p'_m)U_0, \\ V_0 &:= p'_m U_0, \\ O &:= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \end{aligned}$$

The Markov chain  $\{(Q_t, A_t), t \geq 1\}$  is a quasi birth death (QBD) queueing process. If  $\phi V_0 \mathbf{e} < \phi V_2 \mathbf{e}$  where  $\mathbf{e}$  is a  $2 \times 1$  column vector whose elements are all 1, then the QBD process  $\{(Q_t, A_t), t \geq 1\}$  is known to be stable. Hence,  $p_m$  should

satisfy

$$1 - p'_m = \left(1 - \frac{p_m}{N}\right)^M > \lambda. \quad (7)$$

When the QBD process is stable, it has the stationary probability vector  $\pi := (\pi_n, n \geq 0)$  of  $P$  where  $\pi_n := (\pi_{(n,0)}, \pi_{(n,1)})$  for  $n \geq 0$ . Here,  $\pi_n, n \geq 0$ , are given as follows [12].

$$\begin{aligned} \pi_0 &= (\pi_{idle}, 0), \\ \pi_n &= \frac{1}{p'_m} \pi_0 G^n, \quad n \geq 1, \end{aligned}$$

where the matrix  $G$  is the nonnegative minimal solution of the equation

$$G = V_0 + GV_1 + G^2V_2,$$

and  $\pi_{idle}$  is given in (4).

By applying Little's formula, the average packet delay  $D_p$  of the tagged PU is given by

$$\begin{aligned} D_p &:= \frac{E[\lim_{t \rightarrow \infty} Q_t]}{\lambda} \\ &= \frac{1}{\lambda} \sum_{n=1}^{\infty} n \pi_n \mathbf{e} \\ &= \frac{1}{\lambda} \sum_{n=1}^{\infty} \sum_{k=1}^n \pi_0 G^n \mathbf{e} \\ &= \frac{1}{\lambda} \sum_{k=1}^{\infty} \sum_{n=k}^{\infty} \pi_0 G^n \mathbf{e}. \end{aligned}$$

It is well known that the spectral radius of  $G$  is less than 1 when the stability condition holds. In the case, using the fact that,

$$(I - G)^{-1} = \sum_{k=0}^{\infty} G^k,$$

we finally obtain

$$\begin{aligned} D_p &= \frac{1}{\lambda} \sum_{k=1}^{\infty} \pi_0 G^k (I - G)^{-1} \mathbf{e} \\ &= \frac{1}{\lambda} \pi_0 G (I - G)^{-2} \mathbf{e}. \end{aligned}$$

**5.2. Throughput Analysis for an SU.** We now consider the throughput performance of an SU. The throughput  $T_s$  of an SU is defined by

$$T_s := \lim_{t \rightarrow \infty} \frac{E[c(t)]}{t},$$

where  $c(t)$  denotes the number of successfully transmitted packets of the SU until slot  $t$ .

In order to derive the throughput performance of an SU, we tag an arbitrary SU as our reference. Let  $Z_t$  indicate whether the tagged SU transmits a packet at slot  $t$  or not. That is,  $Z_t = 1$  if the tagged SU transmits a packet at slot  $t$ , or  $Z_t = 0$  otherwise. From its definition  $Z_t$  depends on the channel sensing result of

the tagged SU. Let  $C_t$  denote the channel selected by the tagged SU at slot  $t$  and  $Q_t^{(i)}$  ( $i = 1, \dots, N$ ) denote the number of packets at the middle of slot  $t$  in the queue of the  $i$ -th PU. Then the selected channel by the tagged SU is idle at slot  $t$  if  $Q_t^{(C_t)} = 0$  and busy otherwise.

The success of the transmission of the tagged SU depends on the channel selections of all untagged SUs as well as on the state of the selected channel (idle or busy) by the tagged SU, so we need to introduce the channel access process by all untagged SUs at slot  $t$ , which is denoted by  $W_t$ .  $W_t$  is defined by 1 if at least one of untagged SUs transmits its packet through the channel selected by the tagged SU at slot  $t$ , and by 0 otherwise.

Let  $N_t$  be the number of untagged SUs who select the same channel that the tagged SU does. Then  $\{N_t, t \geq 1\}$  are i.i.d. with binomial distribution having parameters  $M-1$  and  $1/N$ . When  $Q_t^{(C_t)} = 0$ , i.e., the selected channel is idle, each untagged SU transmits its packet with probability  $1 - p_f$ . It then follows that

$$\begin{aligned} Pr\{W_t = 0 | Q_t^{(C_t)} = 0\} &= \sum_{k=0}^{M-1} Pr\{W_t = 0 | Q_t^{(C_t)} = 0, N_t = k\} Pr\{N_t = k | Q_t^{(C_t)} = 0\} \\ &= \sum_{k=0}^{M-1} (p_f)^k \binom{M-1}{k} \left(\frac{1}{N}\right)^k \left(\frac{N-1}{N}\right)^{M-1-k} \\ &= \left(1 - \frac{1-p_f}{N}\right)^{M-1}. \end{aligned}$$

Define  $s$  by the probability that the tagged SU successfully transmits its packet at slot  $t$  when the selected channel is idle. We then have

$$\begin{aligned} s &= Pr\{Z_t = 1, W_t = 0 | Q_t^{(C_t)} = 0\} \\ &= Pr\{Z_t = 1 | Q_t^{(C_t)} = 0\} Pr\{W_t = 0 | Q_t^{(C_t)} = 0\} \\ &= (1 - p_f) \left(1 - \frac{1-p_f}{N}\right)^{M-1}. \end{aligned} \tag{8}$$

The second equality holds because SUs independently perform the channel sensing at each slot.

Based on the above observation, we then obtain

$$\begin{aligned} T_s &= \lim_{t \rightarrow \infty} \frac{E[c(t)]}{t} \\ &= \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{i=1}^t Pr\{Z_i = 1, W_i = 0, Q_i^{(C_i)} = 0\} \\ &= \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{i=1}^t Pr\{Z_i = 1, W_i = 0 | Q_i^{(C_i)} = 0\} Pr\{Q_i^{(C_i)} = 0\} \\ &= s \cdot \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{i=1}^t Pr\{Q_i^{(C_i)} = 0\}. \end{aligned} \tag{9}$$

Since the PU network is homogeneous,  $Pr\{Q_i^{(j)} = 0\}$  has the same value for all  $j = 1, \dots, N$ . Thus,

$$\begin{aligned} \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{i=1}^t Pr\{Q_i^{(C_i)} = 0\} &= \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{i=1}^t Pr\{Q_i = 0\} \\ &= \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{i=1}^t (Pr\{(Q_i, A_i) = (0, 0)\} + Pr\{(Q_i, A_i) = (0, 1)\}) \\ &= \pi_{(0,0)} + \pi_{(0,1)} = \pi_{idle}. \end{aligned}$$

The third equality holds because  $\{(Q_t, A_t), t \geq 1\}$  is ergodic under the stability condition. Combining (6), (8), the throughput  $T_s$  of the tagged SU is given by

$$\begin{aligned} T_s &= s \cdot \pi_{idle} \\ &= (1 - p_f) \left(1 - \frac{1 - p_f}{N}\right)^{M-1} \left(1 - \lambda \left(1 - \frac{p_m}{N}\right)^{-M}\right). \end{aligned} \quad (10)$$

## 6. Cross-layer Design and Optimization.

**6.1. Cross-layer Optimization for Secondary Users.** From (10) we see that the imperfect channel sensing affects the throughput performance of an SU. Hence, in order to maximize the throughput performance of an SU, it is important to decide a suitable threshold  $\tau$  in the channel sensing method implemented at the PHY layer, which is a cross-layer optimization of our cognitive radio network. The following proposition guarantees that our cross-layer design optimization is feasible, that is, there exists a threshold  $\tau$  in (1) and (2) (equivalently, the existence of false alarm and misdetection probabilities) that maximizes the throughput of an SU.

**Proposition 1.** *There exists a false alarm probability  $p_f^*$  that maximizes the throughput  $T_s$  of an SU and satisfies the stability condition (7). Furthermore, when  $N \leq M$ ,  $p_f^*$  is in  $[1 - \frac{N}{M}, 1]$ .*

*Proof.* Note that  $p_m = f(p_f)$  is a continuous function of  $p_f$ . From (10), we know that  $T_s$  is also a continuous function of  $p_f$  with a compact domain  $[0, 1]$  (i.e.,  $p_f \in [0, 1]$ ). Hence, there should exist the false alarm probability  $p_f^*$  that maximizes the value of  $T_s$ . Clearly  $p_m^* = f(p_f^*)$  satisfies (7), because  $T_s < 0$  if (7) is violated.

Assuming  $N \leq M$ , we now show that  $p_f^*$  is in  $[1 - \frac{N}{M}, 1]$ . From (6), we know that  $\pi_{idle}$  is a decreasing function in  $p_m$ . Since  $p_m = f(p_f)$  is decreasing in  $p_f$ ,  $\pi_{idle} = \pi_{idle}(p_f)$  is an increasing function in  $p_f$ .

From (8), we observe that  $s = s(p_f)$  increases when  $p_f \in (0, 1 - \frac{N}{M})$  and decreases when  $p_f \in (1 - \frac{N}{M}, 1)$ . We then know that both  $\pi_{idle}(p_f)$  and  $s(p_f)$  are increasing on  $[0, 1 - \frac{N}{M}]$ . Consequently,  $T_s = T_s(p_f) = s(p_f)\pi_{idle}(p_f)$  satisfies that  $T_s(p_f) \leq T_s(1 - \frac{N}{M})$  for all  $p_f \in [0, 1 - \frac{N}{M}]$ . Therefore, the maximum value is achieved at  $p_f \in [1 - \frac{N}{M}, 1]$ .  $\square$

The result in Proposition 1 is very interesting. Without the cross-layer modeling, a smaller false alarm probability provides a better throughput performance of an SU [9]. In particular, the IEEE 802.22 standard recommends that both the false alarm and the misdetection probabilities be less than 0.1. However, when  $N \leq M$ , i.e., the network is crowded, Proposition 1 says that it is not suitable to maintain a small false alarm probability in channel sensing of SUs. Instead, it is recommended

that the false alarm probability  $p_f$  be at least as large as  $1 - \frac{N}{M}$  to maximize the throughput of an SU. Note that the use of small misdetection and false alarm probabilities come from the PHY layer viewpoint to minimize channel sensing errors. Therefore, the requirement in sensing errors should be designed from the cross-layer point of view. In addition, we see from Proposition 1 that the values of  $p_f^*$  and  $p_m^*$  depend the number  $N$  of channels as well as the number  $M$  of SUs.

**6.2. Cross-layer Optimization for both Primary and Secondary Users.** In CR networks, SUs are allowed to access the channels only when the performance of PUs should not be affected by SUs. However, any channel sensing method is imperfect, and hence in the design of channel sensing it is desirable to consider a requirement on the performance of PUs which should not be violated. In addition, we need to maximize the throughput performance of SUs at the same time.

Since we consider the delay performance as the performance metric of PUs in this paper, we consider an optimization problem for the design of the generic channel sensing method to maximize the throughput performance of an SU subject to the constraint on the delay performance of a PU, denoted by  $D_{req}$ , as follows.

$$\begin{aligned} & \max_{p_f} \quad T_s(p_f), \\ & \text{subject to} \quad D_p \leq D_{req}. \end{aligned}$$

We solve the optimization problem numerically to find the optimal value of  $p_f$ , denoted by  $p_{f,D}^*$ . We call the channel sensing method with  $p_{f,D}^*$  the optimal channel sensing method from now on. One interesting issue when we consider the delay requirement  $D_{req}$  in the cross-layer optimization problem is regarding whether there is a difference between  $p_f^*$  and  $p_{f,D}^*$ . We will discuss this issue through numerical and simulation studies in the next subsection.

**6.3. Numerical Results.** In this subsection we provide numerical results based on our cross-layer analysis. We also provide simulation results in order to validate our analytic results. The simulation results are obtained by averaging over 5 independent runs using Matlab, and each run is performed for  $10^6$  slots.

In our simulation, we consider the energy detection method as the channel sensing method, which is widely used in practice because of its low computational and implementation complexities [26]. Since we ignore the channel fading, the channel between a PU and an SU is assumed to be the additive white Gaussian noise channel. Thus, sensing error probabilities of the energy detection method are given by

$$\begin{aligned} p_f &= \Gamma(m, \tau), \\ p_m &= 1 - \Gamma\left(m, \frac{\tau}{1 + \gamma}\right), \end{aligned}$$

where  $m$  denotes the number of signal samples during a channel sensing and  $\gamma$  denotes the average received SNR [24]. Here,  $\Gamma(\cdot, \cdot)$  denotes the regularized incomplete gamma function defined by  $\Gamma(m, \tau) = \frac{1}{\Gamma(m)} \int_{\tau}^{\infty} x^{m-1} e^{-x} dx$  and  $\Gamma(\cdot)$  denotes the gamma function. In our simulation, we use  $m = 15$  and  $\gamma = 0$  dB. The complementary receiver operating characteristic (ROC) curve ( $p_f$  vs.  $p_m$ ) under these parameters is given in Fig. 3. As seen in the figure, it is possible to make both  $p_f$  and  $p_m$  less than 0.1, which is a requirement recommended in the IEEE 802.22 standard [29]. So our parameter values in this subsection makes sense.

To change the false alarm (or misdetection) probability, we change the threshold value  $\tau$  in the channel sensing method. Accordingly, the misdetection and false

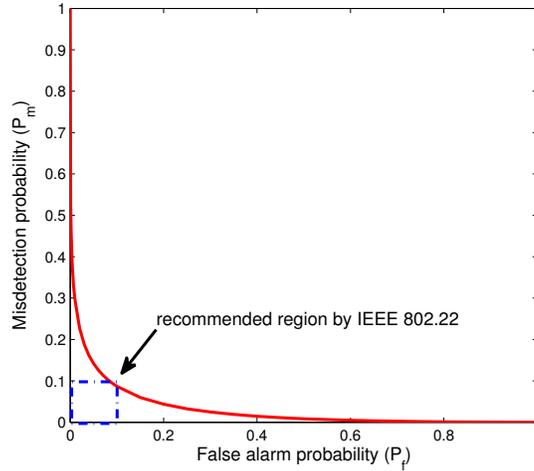


FIGURE 3. The complementary ROC curve.

TABLE 1. The misdetection/false alarm probabilities that maximize the throughput of an SU.

	$N = 23, M = 10$	$N = 31, M = 10$	$N = 10, M = 23$	$N = 10, M = 31$
$p_f^*$	0.0256	0.0168	0.5699	0.6795
$p_m^*$	0.2031	0.2445	0.0057	0.0029

alarm probabilities are simultaneously changed. To show direct relations between the performance metrics and channel sensing errors, we use the false alarm probability  $p_f$  instead of  $\tau$  in the  $x$ -axis of figures. Considering the condition in Proposition 1, we consider two different network scenarios; 1) a sparse network ( $N > M$ ) and 2) a crowded network ( $N \leq M$ ). For the sparse network, we use  $N = 23, M = 10$  and  $N = 31, M = 10$ . For the crowded network, we use  $N = 10, M = 23$  and  $N = 10, M = 31$ . In addition, the transition probabilities of the packet arrival process are given by  $a = 0.2$  and  $b = 0.6$ . Thus, the average packet arrival rate is equal to 0.25.

In Fig. 4, we plots the throughput  $T_s$  of an SU as we change  $p_f$  from 0 to 1. From the figure we see that the analytic and simulation results of the throughput performance are well matched, which validates that our analysis is correct. Recall that  $p_f^*$  and  $p_m^*$  denote the false alarm and misdetection probabilities that maximize the throughput performance of an SU, respectively. The values of  $p_f^*$  and  $p_m^*$  are given in Table 1 for our examples. We see that  $p_f^*$  for the sparse network is relatively small and  $p_f^*$  for the crowded network is relatively large. Accordingly,  $p_m^*$  for the sparse network is relatively large and  $p_m^*$  for the crowded network is relatively small. It is interesting that the values of  $p_f^*$  and  $p_m^*$  in all scenarios do not satisfy the requirement of the false alarm and misdetection probabilities recommended by the IEEE 802.22 standard (which is that both probabilities should be less than 0.1 [29]).

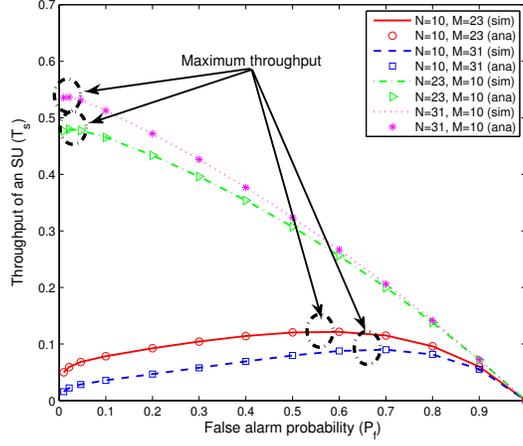


FIGURE 4. The false alarm probability vs. the throughput of an SU.

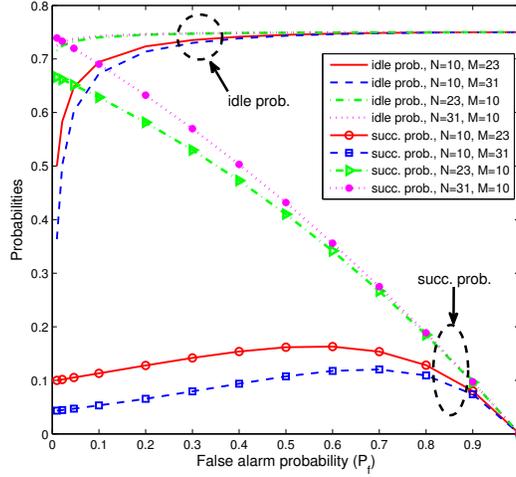


FIGURE 5. The false alarm probability vs. the idle and success probabilities.

When the network is crowded, i.e.,  $M \geq N$ , from Proposition 1, we know the value  $p_f^*$  occurs in  $[1 - \frac{N}{M}, 1]$ . So the lower bound  $1 - \frac{N}{M}$  of  $p_f^*$  cannot be small in the crowded network as seen in Fig. 4. To explain the reason in detail, noting that the throughput of  $T_s$  of an SU is given by  $T_s = s \cdot \pi_{idle}$ , we plot  $s$  and  $\pi_{idle}$  separately in Fig. 5. As seen in the figure,  $\pi_{idle}$  is increasing in  $p_f$  but almost invariant for  $p_f \geq 0.3$ . So the success probability  $s$  becomes dominant for  $p_f \geq 0.3$ , and the value  $p_f^*$  occurs in  $[0.3, 1]$  for the crowded network. This observation implies that, when the network is crowded, the collision avoidance among SUs is more important than the interference alleviation for PUs, because the dominant factor is not  $\pi_{idle}$  but  $s$ . In fact, the false alarm probability  $p_f$  plays the role of limiting the access

of idle channels by SUs, which can alleviate packet collisions between SUs. So the result that  $p_f^*$  is relatively large is quite natural for the crowded network.

Next, we consider the sparse network, i.e.  $M < N$ . From Fig. 5 we see in this case that the change of  $\pi_{idle}$  is almost invariant when  $p_f \geq 0.1$ , but  $s$  significantly decreases as  $p_f$  increases. This implies that the value  $p_f^*$  has to be less than 0.1 as given in Table 1. In conclusion, our cross-layer analysis shows that the false alarm probability should be small (and hence the misdetection probability should be large) for the sparse network, and that the false alarm probability should not be small for the crowded network.

We now investigate the performance of the optimal channel sensing policy with  $p_{f,D}^*$  through numerical examples. To validate our analysis, simulation results are also provided. For numerical and simulation studies, we use the same parameter values as given above unless otherwise mentioned. We consider a delay requirement  $D_{req} = 1.01$ , and we also consider  $D_{req} = \infty$  (no requirement on the delay and hence  $p_{f,D}^* = p_f^*$ ) for comparison purpose.

We know from our observation that the throughput performance of an SU in the sparse network is maximized for relatively large values of  $p_m$ . A large value of  $p_m$  results in a long delay of a PU. So, the consideration of the delay requirement  $D_{req}$  in our optimization problem is expected to be important for the sparse network. On the other hand, the throughput performance of an SU in the crowded network is maximized with relatively small values of  $p_m$ . This implies that the delay requirement  $D_{req}$  is expected to be less important for the crowded network.

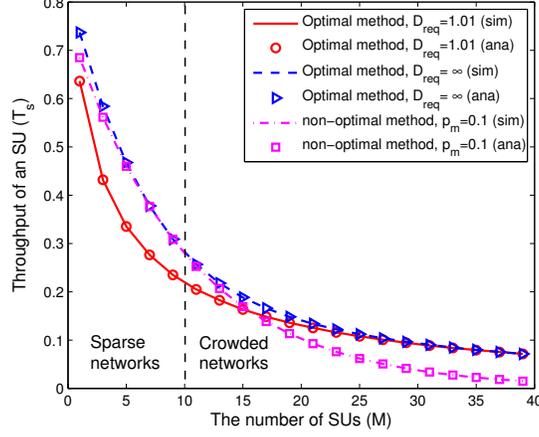
In Fig. 6, the throughput  $T_s$  of an SU and the delay  $D_p$  of a PU are plotted as we change the number  $M$  of SUs from 1 to 40 with a fixed value of  $N = 10$ . From Fig. 6(a), as mentioned above, when the network is sparse, the delay requirement is important in the optimization problem because the difference in  $T_s$  between two cases  $D_{req} = 1.01$  and  $D_{req} = \infty$  is significant. On the other hand, as the number  $M$  of SUs gets large, i.e., as the network gets crowded, the delay requirement does not affect the optimal performance because the difference in  $T_s$  between two cases  $D_{req} = 1.01$  and  $D_{req} = \infty$  becomes negligible. Note that the delay requirements are all satisfied for all networks as seen in Fig. 6(b).

For comparison purpose we also plot the throughput  $T_s$  of an SU in Fig. 6(a) when we use a fixed value of  $p_m = 0.1$ . This channel sensing is denoted by *non-optimal method* in the figure. The reason for the choice of  $p_m = 0.1$  is because it is recommended in the IEEE 802.22 standard [29]. As seen from the figure that the use of the non-optimal method is not suitable for the crowded network because it significantly degrades the throughput performance of SUs. For the sparse networks, the non-optimal method seems to provide better throughput performance of SUs than the optimal method with  $D_{req} = 1.01$ . However, as seen in Fig. 6(b), the non-optimal method violates the delay requirement of PUs.

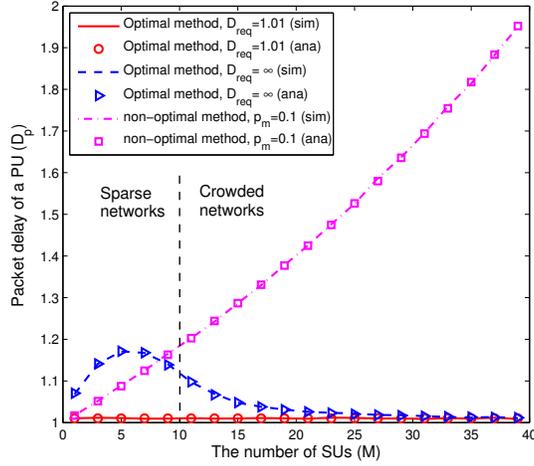
Based on our observation, we conclude that the cross-layer design is important especially for crowded networks where SUs have to use a relatively large value of  $p_f$  in order to optimize the throughput performance of SU.

## 7. Further Discussions.

**7.1. The Impact of Interference in Modeling and Analysis.** The ARQ protocol at PU's MAC layer affects the throughput performance of an SU. That is, when the transmission of a PU is interfered by SUs due to imperfect channel sensing, the ARQ protocol retransmits the interfered packet of the PU, which increases



(a) The throughput of an SU



(b) The average packet delay of a PU

FIGURE 6. The number of SUs vs. the performances of users.

the number of busy slots of the channel and accordingly affects the throughput performance of SUs. To see what happens when we do not consider the interference from SUs due to imperfect channel sensing in the mathematical modeling, we consider a *reference* model where we do not consider the interference from SUs (and hence retransmission by the ARQ protocol) in the channel occupancy model of PUs. Using the same argument as given in Section 5.2, the throughput of the tagged SU in the reference model is given by

$$(1 - p_f) \left(1 - \frac{1 - p_f}{N}\right)^{M-1} (1 - \lambda). \quad (11)$$

Note that the throughput of the tagged SU in the reference model is also of the form given by (9) except the probability that a channel is idle at a slot of being  $1 - \lambda$ . From (11) we see the throughput of an SU in the reference model achieves its maximum value at  $\tilde{p}_f := (1 - \frac{M}{N})^+$ .

We now assume that each SU uses a channel sensing method with the maximum false alarm probability  $\tilde{p}_f$  and its corresponding misdetection probability  $\tilde{p}_m := f(\tilde{p}_f)$ . Let  $\{\tilde{S}_t, t \geq 1\}$  be the resulting service capacity process, and  $\tilde{Q}_t$  be the resulting queueing process of the tagged PU in the reference model. It then follows from the same argument as given in (4) that

$$\begin{aligned} \tilde{Q}_1 &= 0, \quad \tilde{Q}_t = (\tilde{Q}_{t-1} - \tilde{S}_{t-1})^+ + A_t, \text{ for } t \geq 2, \\ Pr\{\tilde{S}_t = 1 | \tilde{Q} > 0\} &= \left(1 - \frac{\tilde{p}_m}{N}\right)^M. \end{aligned}$$

For comparison purpose, we consider the queueing process  $\{Q_t, t \geq 1\}$  given in (3) and the service capacity process  $\{S_t, t \geq 1\}$  with the maximum false alarm probability  $p_f^*$  in Proposition 1 and its corresponding misdetection probability  $p_m^* := f(p_f^*)$ . We have the following proposition which shows the impact of the consideration of interference in the mathematical modeling.

**Proposition 2.** *For the two queueing processes  $\{\tilde{Q}_t, t \geq 1\}$  and  $\{Q_t, t \geq 1\}$ , it satisfies that, for all  $x \geq 0$  and  $t \geq 1$ ,*

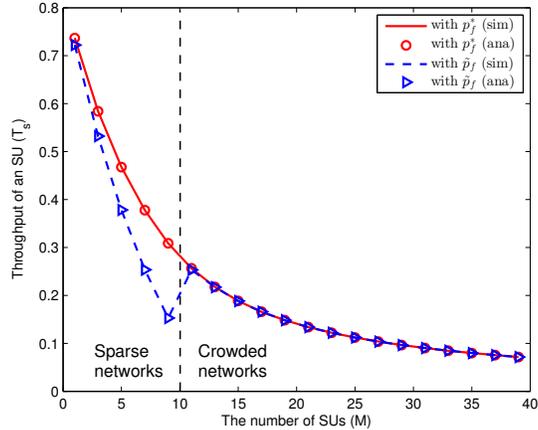
$$Pr\{\tilde{Q}_t > x\} \geq Pr\{Q_t > x\}.$$

Moreover,  $Pr\{\tilde{Q} > x\} \geq Pr\{Q > x\}$  where  $\tilde{Q} = \lim_{t \rightarrow \infty} \tilde{Q}_t$  and  $Q = \lim_{t \rightarrow \infty} Q_t$ .

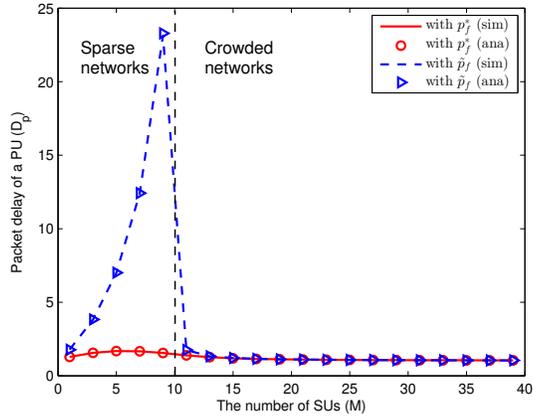
*Proof.* See APPENDIX.  $\square$

Let  $\tilde{D}_p$  denote the average packet delay of the tagged PU obtained from  $\tilde{Q}_t$  and  $D_p^*$  denote the average packet delay of the tagged PU obtained from  $Q_t$ . Little's formula and Proposition 2 show that  $\tilde{D}_p \geq D_p^*$ . Moreover, since the throughput  $T_s$  of an SU is a function of the false alarm probability (i.e.,  $T_s = T_s(p_f)$ ) and  $T_s^* := T_s(p_f^*)$  is the maximum throughput of an SU over all possible values of  $p_f$  by Proposition 1, obviously we have  $\tilde{T}_s := T_s(\tilde{p}_f) \leq T_s^*$ . Hence, we conclude that the interference from an SU due to imperfect channel sensing should be considered in the channel occupancy model of a PU. Otherwise, the optimization analysis results in a misleading conclusion. In the following we validate Proposition 2 through numerical and simulation studies.

Fig. 7 plots the throughput  $T_s$  of an SU and the delay  $D_p$  of a PU as we change the number of SUs from 1 to 40. We consider two scenarios where an SU uses  $p_f^*$  in channel sensing in the first scenario and an SU uses  $\tilde{p}_f$  in channel sensing in the second scenario. In the figure, we denote the scenarios by *with  $p_f^*$*  and *with  $\tilde{p}_f$* . For sparse networks, as we expected from Proposition 2, both  $T_s$  and  $D_p$  are significantly degraded. On the other hand, for crowded networks, the degradation in both  $T_s$  and  $D_p$  is negligible. Note that  $p_f^*$  and  $\tilde{p}_f$  are relatively large in the crowded networks and accordingly  $p_m^*$  and  $\tilde{p}_m$  become almost 0. Consequently, the interference between a PU and an SU becomes negligible in the crowded network. From the observation we conclude that the interference from SUs due to imperfect channel sensing should be considered in the channel occupancy process of a PU, especially for sparse networks. From the figures, we also see our analytic results are well matched with simulation results, which validates our analysis.



(a) The throughput of an SU



(b) The average packet delay of a PU

FIGURE 7. The number of SUs vs. the performances of users.

**7.2. The Impact of Packet Arrival Process.** We see from (10) that the throughput  $T_s$  of an SU does not depend on the individual values of the state transition probabilities  $a$  and  $b$  of the packet arrival process, but depend on the packet arrival rate  $\lambda$ . We thus see that the throughput performance of an SU does not depend on the correlation of packet arrivals of PUs because the correlation of packet arrivals of PUs depends on the values of  $a$  and  $b$ .

We next investigate the impact of the individual values of  $a$  and  $b$  on the delay performance of a PU through numerical examples. Even though the throughput of an SU does not depend on the individual values of  $a$  and  $b$  as mentioned before, the delay  $D_p$  of a PU depends on each value. To this end, we fix the packet arrival rate  $\lambda = 0.25$ . We change the value of  $a$  from 0.05 to 0.3 and accordingly the value of  $b$  is changed from 0.15 to 0.9. In addition, all SUs use  $p_f^*$  in channel sensing to maximize the throughput performance. Fig. 8 shows the delay of a PU for the

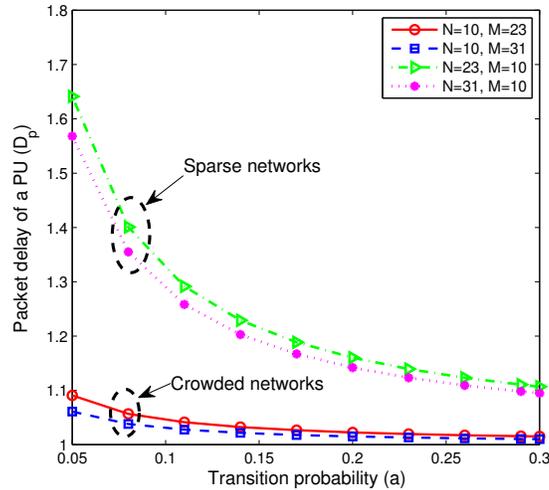


FIGURE 8. The transition probability vs. the average packet delay of a PU.

sparse and crowded networks used in Section 6.3. In the figure, we see that  $D_p$  decreases as  $a$  increases in both networks and the change of  $D_p$  is more significant in the sparse networks than in the crowded networks. This result is explained as follows. Note that the packet arrival process becomes more bursty as the value of  $a$  goes to 0. Hence the delay performance of a PU is worse for a small value of  $a$  in the figure. Note that the misdetection probability  $p_m^*$  in the sparse network is relatively larger than that in the crowded network. Therefore, the delay performance of a PU degrades more significantly in the sparse network.

**8. Conclusions.** In this paper, we considered a cognitive radio network with multiple channels and multiple secondary users. PUs in the network use the ARQ protocol. By considering the characteristics of the ARQ protocol and the direct interference between PUs and SUs, we developed a cross-layer performance model to analyze the delay performance of PUs and the throughput performance of SUs. Using the performance model, we formulated an optimization problem for the design of an optimal channel sensing method that maximizes the throughput performance of SUs while guaranteeing a given delay performance requirement of PUs. From our analysis we showed that the optimal threshold in the channel sensing method depends on the network parameters such as the number of channels and the number of SUs. In addition, we concluded that the use of small false alarm and misdetection probabilities used in many previous works is not suitable and the optimal channel sensing method should be designed from the cross-layer viewpoint.

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## Appendix

**Proof of Proposition 2.** Before starting the proof, we introduce two lemmas that will be used.

**Lemma 8.1.** *Let  $X$  and  $Y$  be random variables such that  $\Pr\{X > x\} \geq \Pr\{Y > x\}$  for all  $x \geq 0$ . Then there exist random variables  $X^\#$  and  $Y^\#$  having the same distribution with  $X$  and  $Y$ , respectively, such that  $X^\# \geq Y^\#$  with probability 1 [22].*

For the proof of Proposition 2, we consider a queueing process  $\{Q_t^{<m>}, t \geq 1\}$  with a new service capacity process  $\{S_t^{<m>}, t \geq 1\}$ , defined by

$$Q_1^{<m>} = 0, Q_t^{<m>} = (Q_{t-1}^{<m>} - S_{t-1}^{<m>})^+ + A_t, \text{ for } t \geq 2,$$

Here, when  $Q_t^{<m>} > 0$ ,  $S_t^{<m>} = S_t$  and when  $Q_t^{<m>} = 0$

$$\begin{aligned} P\{S_t^{<m>} = 1 \mid Q_t^{<m>} = 0\} &= \left(1 - \frac{p_m}{N}\right)^M, \\ P\{S_t^{<m>} = 0 \mid Q_t^{<m>} = 0\} &= 1 - P\{S_t^{<m>} = 1 \mid Q_t^{<m>} = 0\}. \end{aligned}$$

In the case, it is obvious that

$$\begin{aligned} P\{S_t^{<m>} = 1\} &= \left(1 - \frac{p_m}{N}\right)^M, \\ P\{S_t^{<m>} = 0\} &= 1 - P\{S_t^{<m>} = 1\}, \end{aligned}$$

and that  $Q_t^{<m>}$  and  $S_t^{<m>}$  are independent. We then prove the following lemma.

**Lemma 8.2.** *For the two queueing processes  $\{Q_t, t \geq 1\}$  in (3) and  $\{Q_t^{<m>}, t \geq 1\}$  defined above, we have  $Q_t = Q_t^{<m>}$  for all  $t \geq 1$ .*

*Proof.* First,  $Q_1 = Q_1^{<m>} = 0$ . We use the induction argument in the proof. So we assume that  $Q_s = Q_s^{<m>}$  for all  $1 \leq s \leq t$ . It then follows that

$$\begin{aligned} Q_{t+1} &= (Q_t - S_t)^+ + A_{t+1} \\ &= [Q_t - S_t + A_{t+1}]I_{\{Q_t > 0\}} + A_{t+1}I_{\{Q_t = 0\}} \\ &= [Q_t^{<m>} - S_t^{<m>} + A_{t+1}]I_{\{Q_t^{<m>} > 0\}} + A_{t+1}I_{\{Q_t^{<m>} = 0\}} \\ &= (Q_t^{<m>} - S_t^{<m>})^+ + A_{t+1} \\ &= Q_{t+1}^{<m>}, \end{aligned}$$

which completes the proof.  $\square$

Similarly, we can construct  $\tilde{Q}_t^{<m>}$  and  $\tilde{S}_t^{<m>}$  from  $\tilde{Q}_t$  and  $\tilde{S}_t$  where  $\tilde{Q}_t^{<m>}$  and  $\tilde{S}_t^{<m>}$  are independent and  $\tilde{Q}_t^{<m>} = \tilde{Q}_t$  for all  $t \geq 1$ . We now prove Proposition 2.

*Proof of Proposition 2.* In the proof, we use the queueing process  $\{Q_t^{<m>}\}$  instead of  $\{Q_t\}$  because  $Q_t = Q_t^{<m>}$  for all  $t \geq 1$  by Lemma 8.2 and hence both  $Q_t$  and  $Q_t^{<m>}$  have the same distribution function. The benefit of using  $Q_t^{<m>}$  in the proof is that we can use the property that  $P\{S_t^{<m>} = 1\}$  decreases in  $p_m$ .

By abuse of notation, we denote  $\tilde{S}_t^{<m>}$  by *the service capacity of the network* when the misdetection probability is equal to  $\tilde{p}_m = f(\tilde{p}_f)$ . Similarly, we denote  $S_t^{<m>}$  by *the service capacity of the network* when the misdetection probability is equal to  $p_m^* = f(p_f^*)$ . Then we have two queueing processes  $\{\tilde{Q}_t^{<m>}\}$  and  $\{Q_t^{<m>}\}$  given by

$$\begin{aligned} \tilde{Q}_{t+1}^{<m>} &= (\tilde{Q}_t^{<m>} - \tilde{S}_t^{<m>})^+ + A_{t+1}, \\ Q_{t+1}^{<m>} &= (Q_t^{<m>} - S_t^{<m>})^+ + A_{t+1}. \end{aligned}$$

We now assume that the statement is true until a given  $t$ , i.e., for all  $x \geq 0$

$$Pr\{\tilde{Q}_t^{<m>} > x\} \geq Pr\{Q_t^{<m>} > x\}. \quad (12)$$

Recall that  $p_f^* \geq \tilde{p}_f$  when  $N \leq M$  by Proposition 1 and  $p_f^* > \tilde{p}_f = 0$  when  $N > M$ . Hence,  $p_f^* \geq \tilde{p}_f$  for all pairs of  $N$  and  $M$ , and it implies that  $p_m^* = f(p_f^*) \leq f(\tilde{p}_f) =: \tilde{p}_m$ . Since  $p_m^* \leq \tilde{p}_m$ , we have  $Pr\{S_t^{<m>} = 1\} \geq Pr\{\tilde{S}_t^{<m>} = 1\}$ . By Lemma 8.1, there exist random variables  $S_t^\#$  and  $\tilde{S}_t^\#$  having the same distributions with  $S_t^{<m>}$  and  $\tilde{S}_t^{<m>}$ , respectively, such that  $S_t^\# \geq \tilde{S}_t^\#$  with probability 1. Furthermore, from (12) and Lemma 8.1 there exist random variables  $Q_t^\#$  and  $\tilde{Q}_t^\#$  having the same distribution with  $Q_t^{<m>}$  and  $\tilde{Q}_t^{<m>}$ , respectively, such that  $Q_t^\# \leq \tilde{Q}_t^\#$  with probability 1. In addition, it is known that two pairs  $(S_t^\#, \tilde{S}_t^\#)$  and  $(Q_t^\#, \tilde{Q}_t^\#)$  can be generated to be independent. Since  $\tilde{S}_t^{<m>}$  and  $\tilde{Q}_t^{<m>}$  are independent and  $S_t^{<m>}$  and  $Q_t^{<m>}$  are independent, for given  $x \geq 0$ ,

$$\begin{aligned} Pr\{\tilde{Q}_{t+1}^{<m>} > x\} &= Pr\{(\tilde{Q}_t^{<m>} - \tilde{S}_t^{<m>})^+ + A_{t+1} > x\} \\ &= Pr\{(\tilde{Q}_t^\# - \tilde{S}_t^\#)^+ + A_{t+1} > x\} \\ &\geq Pr\{(Q_t^\# - S_t^\#)^+ + A_{t+1} > x\} \\ &= Pr\{(Q_t^{<m>} - S_t^{<m>})^+ + A_{t+1} > x\} \\ &= Pr\{Q_{t+1}^{<m>} > x\}. \end{aligned}$$

Since  $\tilde{Q}_1^{<m>} = Q_1^{<m>} = 0$ , we have  $Pr\{\tilde{Q}_t^{<m>} > x\} \geq Pr\{Q_t^{<m>} > x\}$  for all  $t \geq 1$  and  $x \geq 0$  by induction. Equivalently, from Lemma 8.2 we have shown that  $Pr\{\tilde{Q}_t > x\} \geq Pr\{Q_t > x\}$  for all  $t$ .

Moreover, under the stability condition of the queueing processes  $\{\tilde{Q}_t, t \geq 1\}$  and  $\{Q_t, t \geq 1\}$ , we know that

$$\begin{aligned} Pr\{\tilde{Q} > x\} &= \lim_{t \rightarrow \infty} Pr\{\tilde{Q}_t > x\}, \\ Pr\{Q > x\} &= \lim_{t \rightarrow \infty} Pr\{Q_t > x\}. \end{aligned}$$

Therefore we have  $Pr\{\tilde{Q} > x\} \geq Pr\{Q > x\}$ .  $\square$

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