Analysis of Short Term Fairness and Its Impact on Packet level Performance

Gang Uk Hwang\textsuperscript{a}, Fumio Ishizaki\textsuperscript{b}

\textsuperscript{a}Department of Mathematical Sciences and Telecommunication Engineering Program  
Korea Advanced Institute of Science and Technology (KAIST)  
Daejeon, Republic of Korea  
\textsuperscript{b}Department of Systems Design and Engineering  
Nanzan University, Seto 489-0863, Japan

Abstract

In this paper, we consider the one-bit feedback scheduler with a threshold in a wireless network. Due to its feedback reduction structure, the one-bit feedback scheduler has been studied as one of good candidates implementable in a practical wireless network, and it is important to determine the threshold value of the one-bit feedback scheduler. To solve this problem, network throughput and/or long term fairness are mainly considered in previous studies. However, when we consider the packet level performance such as packet overflow probability, both short term fairness and long term fairness should be considered because both significantly affect the packet level performance. Accordingly, if the scheduler can not provide a higher level of short term fairness to MSs, the packet level performances of MSs are significantly different from MS to MS even when the scheduler provides strict long term fairness.

To see the effect of short term fairness on the packet level performance, we analyze short term fairness and long term fairness of the one-bit feedback scheduler with a given threshold value. We also analyze throughput of each mobile station and network throughput. Based on our analytic results, we investigate the joint effects of short term fairness and network throughput on the packet level performances of MSs. We also show that there is a trade-off between short term fairness and network throughput in the one-bit feedback scheduling through numerical studies.

Email addresses: guhwang@kaist.edu (Gang Uk Hwang), fumio@ieee.org (Fumio Ishizaki)
Key words: One-bit feedback scheduler, packet level performance, short term fairness, long term fairness, network throughput

1. Introduction

In the design of a packet scheduler in a wireless network, maximizing network throughput has been one of important issues and one good solution is to exploit multiuser diversity [1]. By exploiting multiuser diversity, the BS always selects the mobile station (MS) with the best channel condition and accordingly network throughput can be maximized.

However, when MSs have heterogeneous fading channels, providing fairness among multiple MSs is also an important issue. To solve this fairness problem, the proportional fair scheduling has been proposed [2] and/or the normalized received SNR (Signal-to-Noise-Ratio), defined by the ratio of the received SNR over the average of the received SNR, is used. When the normalized received SNR processes of MSs are stochastically identical, the design of a fair scheduler in this case is reduced to that in a homogeneous fading channel case, e.g. [3].

There are two types of fairness - short term fairness and long term fairness [4, 5, 6, 7]. Short term fairness indicates the ability of the scheduler on how equally it can distribute network resources over multiple MSs in a finite size time window. On the other hand, long term fairness indicates the ability of the scheduler on how equally it can distribute network resources over multiple MSs in an infinite size time window. So, while the network throughput is a metric related to the amount of available resources in the network, the fairness indexes are metrics related to the distribution of the network resources to individual MSs. The packet level performances of individual MSs such as packet overflow probability and delay are then affected by both the network throughput and the fairness indexes.

It is known that when we focus on the packet level performances of individual MSs, considering only the long term fairness index as a metric related to the distribution of the network resources is not sufficient. The reason is that the long term fairness index is a quantity related to the first-order statistics for the distribution of the network resources, and the packet level performances of individual MSs are also affected by higher-order statistics. Thus, in order to provide good packet level performances to individual MSs, we should also consider higher-order statistics for the distribution of the network resources. In this paper, as higher-order statistics, we consider the short
term fairness index, which is related to the second-order statistics. We also analyze throughput performances of each MS and the network.

In this paper, we consider the one-bit feedback scheduler which has been widely studies in the open literature [3, 8, 9, 10, 11, 12, 13]. Since the one-bit feedback scheduler exploits multiuser diversity in the wireless network and reduces the amount of feedback information, it is one of good candidates to implement in practice. In addition, it has been known that the one-bit feedback scheduler can achieve almost the same network throughput as the full feedback scheduler [10, 11]. In the one-bit feedback scheduler, a threshold is a priori given. At each time, each MS estimates its received SNR and check if the estimated SNR is greater than or equal to the threshold. MSs whose estimated SNR is greater than or equal to the threshold, can transmit one-bit feedback information to the BS. The scheduler at the BS randomly selects one of MSs which feed back at each time. Due to the structure of the one-bit feedback scheduler, it is very important to determine a suitable threshold value. In most previous studies mentioned above, they consider the information-theoretic capacity which is related with network throughput and long term fairness, and do not consider short term fairness among MSs. In addition, in those studies the authors do not consider the packet level performances of MSs. Later we will show that, when MSs have heterogeneous fading channels, maximizing network throughput does not always guarantee better packet level performances of individual MSs. So, it is important to analyze the joint effects of short term fairness and network throughput on the packet level performances of individual MSs. In this paper, we will show that there is a trade-off between short term fairness and network throughput through numerical examples based on our analysis. It is an implementation issue that which one - short term fairness or network throughput should be considered more importantly, but we believe that our analytic results in this paper can provide a good guideline for the design of a better one-bit feedback scheduler.

The remainder of this paper is organized as follows. In section 2, we give basic assumptions and mathematical models for the wireless fading channels considered in this paper. We also give an explanation of the one-bit feedback scheduler. In section 3, we first analyze short term fairness and long term fairness. We then analyze network throughput and throughput of each MS. In section 4 we provide numerical examples to investigate the joint effects of short term fairness and network throughput. In section 5, we give our conclusions.
2. Preliminaries

2.1. Basic Assumptions

We consider downlink transmission from a base station (BS) to $N$ mobile stations (MSs). Here are basic assumptions regarding to the system modelling.

- Time is divided into physical (PHY) frames of fixed time duration, say, $T_f$, and is indexed by $t$ ($t = 0, 1, 2, \cdots$). Transmissions are performed PHY frame-by-frame.

- The Rayleigh block fading model is used to model the wireless fading channels between the BS and MSs. The received SNR $\gamma_n(t)$ of MS $n$ for every time $t$ is then an exponential random variable with probability density function:

$$p_n(\gamma) = \frac{1}{\bar{\gamma}_n} \exp \left( -\frac{\gamma}{\bar{\gamma}_n} \right),$$

where $\bar{\gamma}_n$ is the average received SNR of MS $n$. We assume that all wireless channels between the BS and MSs are independent.

- Perfect channel estimation is possible at each MS and the received SNR value at each MS is transmitted through an error-free feedback path from the MS to the BS with no delay.

2.2. One-bit Feedback Scheduling and Fairness

The one-bit feedback scheduler performs as follows. At time $t$, MS $n$ estimates its received SNR $\gamma_n(t)$ and check if $\gamma_n(t)$ is greater than or equal to a priori given threshold $\gamma_{th}$. If $\gamma_n(t) \geq \gamma_{th}$, then MS $n$ transmits one-bit feedback information to the BS. Otherwise, MS $n$ does not feed back any information to the BS. The scheduler at the BS randomly selects one of MSs which feed back at time $t$.

It has been known that if the number $N$ of MSs becomes large, the one-bit feedback scheduling can achieve almost the same network throughput as the full feedback scheduling [10, 11]. Here, the full feedback scheduling implies that all MSs feed back their received SNR at each time and the scheduler at the BS selects the MS with the best received SNR value. Thus, the one-bit feedback scheduling is considered as a good candidate to implement in practice because it can significantly reduce the total amount of feedback information, compared to the full feedback scheduling.
While the reduction in the feedback information is important, fairness among MSs is also an important issue in the design of a scheduler. Fairness of a scheduler is closely related with the wireless channel access frequency of individual MSs. That is, if the channel access frequencies of all MSs are the same, we say the scheduler provides strict fairness among MSs. There are two types of fairness - short term fairness and long term fairness. Short term fairness indicates the ability of the scheduler on how equally it can distribute time slots over multiple MSs in a finite size time window. On the other hand, long term fairness indicates the ability of the scheduler on how equally it can distribute time slots over multiple MSs in an infinite size time window. In fact, long term fairness is closely related with the first-order statistic of the scheduler’s behavior. We will later give an exact definition of fairness. Obviously, the performance of the scheduler at the packet level is affected by short term fairness as well as long term fairness. This is because short term fairness is related with the second-order statistic of the scheduler’s behavior. Hence, considering only long term fairness is not sufficient in the design of a fair scheduler. For this reason, we consider and analyze short term fairness as well as long term fairness in this paper.

2.3. Wireless Channel Model

The wireless channel state of each MS is either 0 or 1 based on the threshold $\gamma_{th}$. That is, if the received SNR $\gamma_{n}(t)$ of MS $n$ is greater than or equal to $\gamma_{th}$, the wireless channel state $s_{n}(t)$ of MS $n$ is equal to 1 at time $t$. Otherwise, i.e., $\gamma_{n}(t) < \gamma_{th}$, the wireless channel state $s_{n}(t)$ of MS $n$ is equal to 0 at time $t$. Then, similarly to [14], the wireless channel of MS $n$ can be modelled by a 2-state Markov chain with transition probability matrix $Q_{n}$ as follows. We first consider the level crossing rate $L_{n}(\gamma_{th})$ of the received SNR $\gamma_{n}(t)$ at $\gamma_{th}$, given by [15]

$$L_{n}(\gamma_{th}) = \sqrt{\frac{2\pi\gamma_{th}}{\gamma_{n}}} f_{n,d} \exp \left(-\frac{\gamma_{th}}{\gamma_{n}}\right).$$

(2)

Here, $f_{n,d}$ denotes the mobility-induced Doppler spread of MS $n$.

Let $q_{n,i}$ ($i = 0, 1$) denote the stationary probability that the 2-state
Markov chain \( s_n(t) \) is in state \( i \). Then, it is given by
\[
q_{n,0} = \int_0^{\gamma_{th}} p_n(\gamma) d\gamma = 1 - \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_n}\right),
\]
\[
q_{n,1} = \int_{\gamma_{th}}^\infty p_n(\gamma) d\gamma = \exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_n}\right).
\]

Using the argument given in [14], we see that the transition probability matrix \( Q_n \) of \( s_n(t) \) is given by
\[
Q_n = \begin{pmatrix}
\beta_n & 1 - \beta_n \\
1 - \alpha_n & \alpha_n
\end{pmatrix}.
\]
Here, the transition probability from state 1 (resp. state 0) to state 0 (resp. state 1) is given by \( 1 - \alpha_n \) (resp. \( 1 - \beta_n \)), where \( 0 < \alpha_n, \beta_n < 1 \). The values of \( \alpha_n \) and \( \beta_n \) are given by
\[
1 - \beta_n = \frac{L_n(\gamma_{th})T_f}{q_{n,0}}, \quad 1 - \alpha_n = \frac{L_n(\gamma_{th})T_f}{q_{n,1}}.
\]

Next, let \( S(t) \) be the wireless channel state vector of all MSs, that is,
\[
S(t) := (s_1(t), s_2(t), \cdots, s_N(t)).
\]
Since all \( s_n(t), 1 \leq n \leq N \) are independent Markov chains, \( S(t) \) is also a Markov chain with state space
\[
\mathcal{S} := \{(i_1, \cdots, i_N) : i_k = 0 \text{ or } 1, 1 \leq k \leq N\}
\]
and transition probability matrix
\[
Q_1 \otimes Q_2 \otimes \cdots \otimes Q_n
\]
where \( \otimes \) denotes the Kronecker product. We assume that the wireless channel process \( S(t) \) is stationary. Then, the stationary probability vector \( \pi \) of \( S(t) \) is given by
\[
\pi = q_1 \otimes q_2 \otimes \cdots \otimes q_N
\]
where \( q_n := (q_{n,0}, q_{n,1}) \).

There are \( M := |\mathcal{S}| = 2^N \) states in \( \mathcal{S} \). The states of \( S(t) \) are ordered in the lexicographic order, so that notations \( P = (p_{ij}) \) and \( \pi = (\pi_i) \) are not ambiguous. That is, \( p_{ij} \) denotes the \((i, j)\)-th element of \( P \) and \( \pi_i \) is the \( i \)-th element of \( \pi \). Further, by the abuse of notation we use \( S(t) \) instead of \( S(t) \), if needed, in our analysis later where \( S(t) = i \) means that the wireless channel state \( S(t) \) is equal to the \( i \)-th state in \( \mathcal{S} \) at time \( t \).
3. Performance Analysis

3.1. Fairness Analysis

We consider a window \([0, t]\) and let \(I_n(t)\) be the number of slots assigned to MS \(n\) during the window \([0, t]\). The short term fairness index is defined by [4]

\[
SF(t) := \left( \frac{\sum_{n=1}^{N} E[I_n(t)]}{N \sum_{n=1}^{N} E[I_n(t)^2]} \right)^2.
\]

The long term fairness index is then defined by

\[
SF(\infty) := \lim_{t \to \infty} SF(t).
\]

To compute the short term fairness index of the scheduler, we define

\[
\psi_{n,i}(\theta, t) := E[e^{\theta I_n(t)}|S(1) = i],
\]

\[
\Psi_n(\theta, t) := (\psi_{n,1}(\theta, t), \cdots, \psi_{n,M}(\theta, t))^*,
\]

\[
\phi_{n,i}(\theta) := \psi_{n,i}(\theta,1),
\]

\[
\Phi_n := \text{diag}(\phi_{n,1}(\theta), \cdots, \phi_{n,M}(\theta))
\]

where the superscript * denotes the transpose of the row vector. Then we get the following:

\[
\psi_{n,i}(\theta, t) = E[e^{\theta I_n(t)}|S(1) = i] = \sum_j E[e^{\theta |I_n(t)-I_n(1)|}]E[I_n(1)|S(1) = i]P\{S(2) = j|S(1) = i\}
\]

\[
= \phi_{n,i}(\theta) \sum_j E[e^{\theta |I_n(t-1)|}]E[I_n(1)|S(1) = j]p_{ij}
\]

It then follows that

\[
\Psi_n(\theta, t) = \Phi_n(\theta)\Psi_n(\theta, t-1)
\]

\[
= [\phi_n(\theta)P]^{t-1}\psi_n(\theta, 1)
\]

\[
= [\phi_n(\theta)P]^{t-1}\phi_n(\theta)e
\]

where \(e\) is an \(M \times 1\) column vector whose elements are all equal to 1.
Hence, it follows that

\[
E[e^\theta I_n(t)] = \pi \phi_n(\theta, t) \\
= \pi [\phi_n(\theta)P]^{t-1} \phi_n(\theta)e \\
= \pi [\phi_n(\theta)P]^{t-1} \phi_n(\theta)Pe \\
= \pi [\phi_n(\theta)P]^t e.
\]

In the above derivation, \( \phi_n(\theta) \) can be computed by its definition as follows. We consider the case for \( S(1) = i \) and let \( (i_1, i_2, \ldots, i_N) \) be the \( i \)-th state in the state space \( S \) of \( \{S_n(t)\} \). That is, we have \( S_n(1) = (i_1, i_2, \ldots, i_N) \) in this case. Then, \( \sum_{k=1}^{N} i_k \) is the number of MSs that feed back at time 1 because \( i_k \) is equal to 1 if MS \( k \) feeds back at time 1, and is equal to 0 otherwise. Since the scheduler randomly selects one of MSs who feed back, it follows that

\[
\phi_{n,i}(\theta) := E[e^{\theta I_n(1)} | S(1) = i] \\
= \left\{ \begin{array}{ll}
\frac{1}{\sum_{k=1}^{N} i_k} e^{\theta} + \left(1 - \frac{1}{\sum_{k=1}^{N} i_k}\right), & \text{if } \sum_{k=1}^{N} i_k > 0, \\
1, & \text{if } \sum_{k=1}^{N} i_k = 0,
\end{array} \right.
\]

and \( \phi_{n,i}(\theta) \) given above is the \( i \)-th element of \( \phi_n(\theta) \).

From (4), we can compute \( E[I_n(t)], \; E[I_n(t)^2] \) and the short term fairness index as given in the following theorem.

**Theorem 1.** The first and second moments of \( I_n(t) \) are given by

\[
E[I_n(t)] = t\pi \phi'_n(0)e, \\
E[I_n(t)^2] = t\pi \phi''(0)e + 2\pi \phi'_n(0) \left[ \sum_{s=1}^{t-1} \sum_{m=0}^{s-1} P^{s-m} \right] \phi'_n(0)e.
\]

Then, the short term fairness index of the scheduler is given by

\[
SF(t) = \frac{t^2 \left[ \sum_{n=1}^{N} \pi \phi'_n(0)e \right]^2}{Nt\pi \phi''(0)e + \sum_{n=1}^{N} 2N \pi \phi'_n(0) \left[ \sum_{s=1}^{t-1} \sum_{m=0}^{s-1} P^{s-m} \right] \phi'_n(0)e}
\]

The proof of Theorem 1 is given in the Appendix. From Theorem 1, we
have a recursive relation for $E[I_n(t)^2]$ as follows:

$$E[I_n(t+1)^2] = (t+1)\pi \phi''(0)e + 2\pi \phi'_n(0) \left[ \sum_{s=1}^{t} \sum_{m=0}^{s-1} P^{s-m} \right] \phi'(0)e$$

$$= t\pi \phi''(0)e + 2\pi \phi'_n(0) \left[ \sum_{s=1}^{t-1} \sum_{m=0}^{s-1} P^{s-m} \right] \phi'_n(0)e$$

$$+ \pi \phi''(0)e + 2\pi \phi'_n(0) \left[ \sum_{m=0}^{t-1} P^{t-m} \right] \phi'_n(0)e$$

$$= E[I_n(t)^2] + \pi \phi''(0)e + 2\pi \phi'_n(0) \left[ \sum_{m=1}^{t} P^m \right] \phi'_n(0)e.$$

Hence, we have the following corollary which will be useful in numerical studies later.

**Corollary 1.** \{E[I_n(t)^2]\} satisfies the following recursion: For $t \geq 1$

$$E[I_n(t+1)^2] = E[I_n(t)^2] + \pi \phi''(0)e + 2\pi \phi'_n(0) \left[ \sum_{s=1}^{t} P^s \right] \phi'_n(0)e$$

where $E[I_n(1)^2] = E[I_n(1)] = \pi \phi'_n(0)e$.

To obtain the long term fairness index $SF(\infty)$ of the scheduler, we need the following lemma.

**Lemma 1.**

$$\lim_{t \to \infty} \frac{1}{t^2} \sum_{s=1}^{t} \sum_{m=0}^{s-1} P^{s-m} = \frac{1}{2} e \pi.$$

The proof of Lemma 1 is given in the Appendix.

**Theorem 2.** The long term fairness index of the scheduler is given by

$$SF(\infty) = \frac{\left[ \sum_{n=1}^{N} \pi \phi'_n(0)e \right]^2}{N \sum_{n=1}^{N} [\pi \phi'_n(0)e]^2}.$$
From Theorem 2, we see that the long term fairness index depends only on the first-order statistic provided to each MS such as the average numbers of slots assigned to individual MSs. In addition, when the average numbers of slots assigned to individual MSs are all equal, the long term fairness index $SF(\infty)$ is give by

$$SF(\infty) = \frac{\left[\sum_{n=1}^{N} \pi \phi_n'(0)e\right]^2}{N \sum_{n=1}^{N} [\pi \phi_n'(0)e]^2}$$

$$= \frac{[N \pi \phi_1'(0)e]^2}{N^2 [\pi \phi_1'(0)e]^2}$$

$$= 1.$$

That is, we have the following corollary.

**Corollary 2.** When $\pi \phi_n'(0)e$ are all equal for $1 \leq n \leq N$, the long term fairness index becomes 1, i.e., $SF(\infty) = 1$.

In the Rayleigh fading model, recall that the probability density function of the received SNR $\gamma_n(t)$ of MS $n$ is completely determined by the average SNR $\bar{\gamma}_n$. So, if the average SNR values $\bar{\gamma}_n$ are all equal, the long term fairness index is 1. However, the packet level performance of each MS, e.g., the packet overflow probability, depends significantly on the correlation in its wireless fading channel. Accordingly, even though the long term fairness index $SF(\infty)$ is equal to 1, it does not imply that the scheduler provides the same packet level performance to each MS. We will see this later in section 4. For this reason, the short term fairness index $SF(t)$ is important in the design of a fair scheduler, especially when individual MSs have heterogeneous wireless fading channels.

### 3.2. Throughput Analysis

Even though the short term fairness is important to improve performances of individual MSs, throughput is also an important performance metric. For instance, assume that a scheduler can provide good short term fairness but bad throughput performance. In this case, since the throughput performance is bad, the scheduler may not guarantee good packet level performance.

We consider a window $[0, t]$, and let $T_n(t)$ be the number of successfully transmitted packets for MS $n$ during the window $[0, t]$. Then network
throughput $T_{network}$ during a window $[0, t]$ is given by

$$T_{network} := \frac{1}{t} \sum_{n=1}^{N} E[T_n(t)].$$

In addition, the throughput of MS $n$, denoted by $T_{ave,n}$ is given by

$$T_{ave,n} = \frac{1}{t} E[T_n(t)]. \quad (5)$$

To compute $E[T_n(t)]$, we assume that each MS transmits one packet in a time slot, if it is selected by the scheduler, by using the same modulation scheme and BPSK$^1$ is used in this paper. In this case, the packet error rate $\text{PER}_n(\gamma)$ of MS $n$ when $\gamma_n(t) = \gamma$ is given by [16]

$$\text{PER}_n(\gamma) \approx \begin{cases} 1 & (0 < \gamma < \gamma_{bd}), \\ a \exp(-g\gamma) & (\gamma \geq \gamma_{bd}), \end{cases} \quad (6)$$

where we use $a = 67.7328$, $g = 0.9819$, and $\gamma_{bd} = 6.3281$ (dB). The above values of $a$, $g$, and $\gamma_{bd}$ are obtained when the packet size is equal to 1080 bits (see, e.g., [16]).

We then compute the packet error probability $r_n$ when MS $n$ is selected as follows. When $\gamma_{bd} \geq \gamma_{th}$,

$$r_n = \frac{1}{q_{n,1}} \left[ \int_{\gamma_{th}}^{\gamma_{bd}} p_n(\gamma) d\gamma + \int_{\gamma_{bd}}^{\infty} a \exp(-g\gamma)p_n(\gamma) d\gamma \right].$$

When $\gamma_{bd} < \gamma_{th}$,

$$r_n = \frac{1}{q_{n,1}} \int_{\gamma_{th}}^{\infty} a \exp(-g\gamma)p_n(\gamma) d\gamma.$$

Using a similar argument as in the derivation of $E[I_n(t)]$, we get

$$E[T_n(t)] = t \pi \tilde{\phi}_n(0) e \quad (7)$$

$^1$Note that the analysis of this paper can be also applied when we assume another modulation scheme.
where

\[ \tilde{\phi}_n(\theta) := \text{diag}(\tilde{\phi}_{n,1}(\theta), \ldots, \tilde{\phi}_{n,M}(\theta)), \]

\[ \tilde{\phi}_{n,i}(\theta) := E[e^{\theta T_n(1)} | S(1) = i] \]

\[ = \left\{ \begin{array}{ll}
\frac{1 - r_{ij}}{\sum_{k=1}^{N} i_k} e^{\theta} + \left(1 - \frac{1 - r_{ij}}{\sum_{k=1}^{N} i_k}\right), & \text{if } \sum_{k=1}^{N} i_k > 0, \\
1, & \text{if } \sum_{k=1}^{N} i_k = 0.
\end{array} \right. \]

Here, \((i_1, \ldots, i_N)\) denotes the \(i\)-th state in the state space \(S\), so that \(S(1) = i\) implies that \(S(1) = (i_1, \ldots, i_N)\).

Combining equations (5) and (7), we obtain the throughput of MS \(n\) and the network throughput as follows:

\[ T_{\text{ave},n} = \pi \tilde{\phi}_n'(0) e, \quad (8) \]

\[ T_{\text{network}} = \sum_{n=1}^{N} T_{\text{ave},n} = \sum_{n=1}^{N} \pi \tilde{\phi}_n'(0) e. \quad (9) \]

3.3. Design of the one-bit feedback scheduler

When we consider the packet level performances of MSs, we will see in section 4 that both short term fairness and network throughput significantly affect the performances of MSs. So, by considering them simultaneously, we can formulate the following problem (P): For a given required short term fairness index \(R_{\text{STF}}\) and a predetermined window of size \(t\), the threshold value \(\gamma_{th}\) is determined by solving

\[ (P) \max_{\gamma_{th}} T_{\text{network}} \]

\[ \text{s.t. } SF(t) \geq R_{\text{STF}}. \]

We can solve the problem (P) by using our analytic results given in section 3. In section 4, we will provide some examples for which the solutions \(\gamma_{th}\) of the above problem (P) are given.

4. Numerical Results and Discussions

In this section, using our analysis we investigate the performance behavior of the scheduler as we change the threshold \(\gamma_{th}\). We also investigate the effect of heterogeneous wireless channel conditions of MSs on the performance of the scheduler.
In the first example, we check if the short term fairness affects the packet level performance. For this, we consider a wireless network with two MSs, say, MS 1 and MS 2. For comparison purpose, both MSs have the same average SNR 15 dB, but the Doppler frequencies $f_d$ of MS 1 and MS 2 are 10 and 70, respectively. Table 1 shows the numerical results for our first example. In Table 1, we provide $SF(10)$, $SF(150)$ and $SF(\infty)$. Since the Doppler frequency of MS 1 is lower than that of MS 2, the wireless channel process of MS 1 is more correlated than that of MS 2. Thus, the short term fairness index $SF(t)$ is less than 1 for a finite window of size $t$ as seen in Table 1. However, since both MSs have the same average SNR, the long term fairness is equal to 1. In fact, as we increase the window size $t$, we see that $SF(t)$ converges to 1 as seen in Table 1.

From equation (8) we see that all MSs have the same throughput. Accordingly, the throughput of each MS is one half of $T_{network}$. As seen in Table 1, the short term fairness index strictly decreases as we increase the threshold $\gamma_{th}$. On the other hand, the network throughput increases until $\gamma_{th} = 5.49$ as we increase the threshold $\gamma_{th}$.

To see the effect of the short term fairness index on the packet level performance we simulate our wireless network with one-bit feedback scheduling where we use the wireless fading model proposed in [17]. We use $T_f = 0.001$ (second) as one slot time. We assume that there are two FIFO (First-In-First-Out) queues at the BS, each of which is assigned to each MS. We estimate the packet overflow probability from the simulation. In the simulation, we assume that the packet arrival process to each MS is stochastically identical and is according to a Poisson process with rate 0.35 (packets/slot). The simulation results are plotted in Figure 1. Since the value of $SF(t)$ is the largest when $\gamma_{th} = 4.29$ in the table, we use $\gamma_{th} = 4.29$ in simulation and the results are plotted in Figure 1a. On the other hand, since the network throughput is maximized when $\gamma_{th} = 5.49$, we use $\gamma_{th} = 5.49$ in simulation and the results are plotted in Figure 1b. Comparing Figure 1a and 1b, we see that the short term fairness index affects the packet overflow probability and the packet overflow probabilities of two MSs are closer in Figure 1a (for which $SF(t)$ is large) than those in Figure 1b (for which $SF(t)$ is small). In addition, since $\gamma_{th}$ used in Figure 1b provides higher network throughput, the packet overflow probabilities in Figure 1b is smaller than those in Figure 1a. This shows a trade-off between short term fairness and network throughput in the one-bit feedback scheduling. In addition, we see that, even though the long term fairness index $SF(\infty)$ is equal to 1, the one-bit feed back scheduler
Table 1: Fairness Index and Throughput (Example 1)

<table>
<thead>
<tr>
<th>$\gamma_{th}$</th>
<th>$SF(10)$</th>
<th>$SF(150)$</th>
<th>$SF(\infty)$</th>
<th>$T_{network}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.29</td>
<td>0.8309</td>
<td>0.9742</td>
<td>1</td>
<td>0.9532</td>
</tr>
<tr>
<td>4.49</td>
<td>0.8270</td>
<td>0.9730</td>
<td>1</td>
<td>0.9573</td>
</tr>
<tr>
<td>4.69</td>
<td>0.8232</td>
<td>0.9718</td>
<td>1</td>
<td>0.9603</td>
</tr>
<tr>
<td>4.89</td>
<td>0.8193</td>
<td>0.9706</td>
<td>1</td>
<td>0.9625</td>
</tr>
<tr>
<td>5.09</td>
<td>0.8155</td>
<td>0.9694</td>
<td>1</td>
<td>0.9640</td>
</tr>
<tr>
<td>5.29</td>
<td>0.8117</td>
<td>0.9681</td>
<td>1</td>
<td>0.9648</td>
</tr>
<tr>
<td>5.49</td>
<td>0.8079</td>
<td>0.9669</td>
<td>1</td>
<td>0.9652</td>
</tr>
<tr>
<td>5.69</td>
<td>0.8042</td>
<td>0.9656</td>
<td>1</td>
<td>0.9652</td>
</tr>
<tr>
<td>5.89</td>
<td>0.8004</td>
<td>0.9644</td>
<td>1</td>
<td>0.9648</td>
</tr>
</tbody>
</table>

does not provide fairness in the packet level performance between two MSs.

For this first example, we next consider the problem (P) given in section 3.3. We assume that the required value of the short term fairness index is given by $R_{STF} = 0.81$ for a window size $t = 10$. Then, since $T_{network}$ decreases as $\gamma_{th}$ increases as seen in Table 1, the solution $\gamma_{th}$ to the problem (P) for this example is given by $\gamma_{th} = 5.3$.

In the second example, we consider 6 MSs and each has different wireless fading channel. The respective average SNR values of MSs are given by 10, 15, 10, 15, 10, 15 (dB) and the respective Doppler frequencies of MSs are given by 10, 10, 35, 35, 70, 70. In Table 2 we give the short and long term fairness indexes and network throughput $T_{network}$ given in equation (9). As in the first example, $SF(t)$ decreases as we increase the value of $\gamma_{th}$. Similarly, $T_{network}$ increases until $\gamma_{th} = 8.29$ as we increase the value of $\gamma_{th}$. Regarding the long term fairness index, $SF(\infty)$ is strictly less than 1 because all MSs have different wireless fading channels. To see the effect of $SF(t)$ on the packet overflow probability, we simulate our wireless network where the arrival processes to MSs are stochastically identical and are according to a Poisson process with arrival rate 0.1 (packets/slot). The simulation results are given in Figure 2. In Figure 2a, we plot the results when we use $\gamma_{th} = 4.29$ for which the value of $SF(10)$ is the largest in the table. In Figure 2b, we plot the results when we use $\gamma_{th} = 8.29$ for which $T_{network}$ is maximized. As in Figure 1, packet overflow probabilities of MSs are closer in Figure 2a than
those in Figure 2b. This also verifies that $SF(t)$ affects the packet level performance. In addition, it is very interesting to see that the packet overflow probabilities of MS 1, 3, 5 in Figure 2b are worse than those in Figure 2a, even though $\gamma_{th} = 8.29$ used in Figure 2b maximizes network throughput $T_{network}$. The reason for this is that, even though $\gamma_{th} = 8.29$ provides the maximum network throughput, it cannot provide the maximum individual throughput $T_{ave,n}$. In fact, we see that

$$(T_{ave,1}, \cdots, T_{ave,6}) = (0.1250, 0.1895, 0.1250, 0.1895, 0.1250, 0.1895)$$

when $\gamma_{th} = 4.29$. However, when $\gamma_{th} = 8.29$ we have

$$(T_{ave,1}, \cdots, T_{ave,6}) = (0.1123, 0.2199, 0.1123, 0.2199, 0.1123, 0.2199).$$

Accordingly, the packet overflow probabilities for MS 1, 3, 5 are worse when we use $\gamma_{th} = 8.29$. In addition, we see that the difference in the values of $T_{ave,n}$ in Figure 2a is smaller than that in Figure 2b. This means that the use of $\gamma_{th} = 4.29$ provide a better packet level performance. As seen in this example, we see that maximizing network throughput does not always guarantee the improvement in the packet level performance for individual MSs with heterogeneous wireless fading channels.

For this second example, we can also consider the problem (P) given in section 3.3. Here, the required value of the short term fairness index is given...
by $R_{STF} = 0.55$ for a window size $t = 10$. Then, by using our analytic results in section 3, the solution $\gamma_{th}$ to the problem (P) for this example is given by $\gamma_{th} = 6.2$.

In the third example, we examine the effect of the correlation in the wireless fading channel on the short term fairness index. For this, we consider 5 MSs and all MSs have homogeneous fading channels with average SNR 15 (dB). We change the Doppler frequency from 10 to 100 and the results of $SF(10)$ are plotted in Figure 3. From the figure, we see that the short term fairness index $SF(t)$ decreases as $\gamma_{th}$ increases. However, when the wireless fading channel is strongly correlated such as the Doppler frequency is 10 or 50, the short term fairness index $SF(t)$ is more sensitive to $\gamma_{th}$. As the wireless fading channel becomes less correlated, e.g., the Doppler frequency is 100, the short term fairness index $SF(t)$ is less sensitive to $\gamma_{th}$.

From Figure 3, we also see that the short term fairness index $SF(t)$ increases as the fading channel gets less correlated. The reason for this is explained as follows. When the fading channel of each MS is more correlated, the MS is more likely to be selected during the window of interest once it has good channel condition at the initial time of the window. This causes the decrease in the short term fairness index. On the other hand, if the fading
channel is less correlated, each MS is more equally likely to be selected during the window of interest regardless of the channel state at the initial time of the window. This results in the increase in the short term fairness index.

In the last example, we examine the effect of the average SNR on the short term fairness index. In this example, we consider 5 MSs with homogeneous fading channel with Doppler frequency $f_d = 10$. We consider the average SNR values 10, 15, 20 dB. In Figure 4, we plot the resulting values of $SF(10)$ when we change the value of $\gamma_{th}$. As seen in the figure, as the fading channel condition becomes better, the short term fairness index increases and is less sensitive to $\gamma_{th}$. This is because MSs are more likely to be in good channel state, i.e., state 1 of the Markov chain irrespective of the value $\gamma_{th}$ when the average SNR is large. So, almost all MSs can feed back one-bit information during the window of interest and they can have almost equal chances to be selected due to the random selection by the scheduler.

From our numerical examples, we observe that both short term fairness and network throughput significantly affect the packet level performances of MSs and they should be considered in the design of the one-bit feedback scheduler.

Figure 2: Packet overflow probability vs. Threshold $\gamma_{th}$ (Example 2)
The value of $\gamma$th
The short term fairness index $SF(10)$

Figure 3: The effect of the correlation on $SF(t)$

The value of $\gamma$th
The short term fairness index $SF(10)$
average SNR = 10 dB
average SNR = 15 dB
average SNR = 20 dB

Figure 4: The effect of average SNR on $SF(t)$
5. Conclusions

In this paper, we consider the one-bit feedback scheduler in a wireless network and analyze short term fairness, long term fairness and throughput. Based on our analysis, we investigate the joint effects of short term fairness and network throughput. We find that there is a trade-off between short term fairness and network throughput for the one-bit feedback scheduler. We also find that maximizing network throughput does not always guarantee the improvement in the packet level performance for individual MSs with heterogeneous wireless fading channels. Therefore, we conclude that, when the MSs have heterogeneous wireless fading channels, both short term fairness and network throughput should be considered in the design of the one-bit feedback scheduler.

Acknowledgment

This work was supported by the Korea Research Foundation (KRF) grant funded by the Korea government (MEST) (No. 2009-0072665).

Appendix

Proof of Theorem 1

From equation (4), by differentiating both sides we get

$$\frac{d}{d\theta}E[e^{\theta I_n(t)}] = \pi \sum_{s=0}^{t-1} \left[ \phi_n(\theta)P \right]^s \left[ \phi_n'(\theta)P \right] \left[ \phi_n(\theta)P \right]^{t-s-1} e. \tag{10}$$

Letting $\theta = 0$ in equation (10) and using the fact that

$$\phi_n(0) = I, \quad \pi P = \pi, \quad Pe = e$$

where $I$ denotes the identity matrix, we get

$$E[I_n(t)] = \pi \sum_{s=0}^{t-1} \left[ \phi_n(0)P \right]^s \left[ \phi_n'(0)P \right] \left[ \phi_n(0)P \right]^{t-s-1} e$$

$$= \pi \sum_{s=0}^{t-1} P^s \left[ \phi_n'(0)P \right] P^{t-s-1} e$$

$$= \pi \sum_{s=0}^{t-1} \phi_n'(0) e$$

$$= t \pi \phi_n'(0) e.$$
Next, by differentiating both sides of equation (10), we get

\[
\frac{d^2}{d\theta^2} E[e^{\theta I_n(t)}] = \pi \sum_{s=0}^{t-1} \left\{ \sum_{m=0}^{s-1} [\phi_n(\theta)P]^m \phi_n'(\theta)P [\phi_n(\theta)P]^{s-m-1} [\phi_n'(\theta)P] [\phi_n(\theta)P]^{t-s-1} e + [\phi_n(\theta)P]^s \phi_n''(\theta)P [\phi_n(\theta)P]^{t-s-1} e \right\}
\]

\[
[\phi_n(\theta)P]^s \phi_n'(\theta)P \sum_{m=0}^{t-s-2} [\phi_n(\theta)P]^m \phi_n'(\theta)P [\phi_n(\theta)P]^{t-s-m-2} e \right\}
\]

(11)

Letting \( \theta = 0 \) in equation (11) yields

\[
E[I_n(t)^2] = \pi \sum_{s=0}^{t-1} \left\{ \sum_{m=0}^{s-1} P^m \phi_n'(0)P^{s-m} \phi_n'(0)e + P^s \phi_n''(0)e + P^s \phi_n'(0)P \sum_{m=0}^{t-s-2} P^m \phi_n'(0)e \right\}
\]

= \sum_{s=0}^{t-1} \left\{ \sum_{m=0}^{s-1} \pi \phi_n'(0)P^{s-m} \phi_n'(0)e + \pi \phi_n''(0)e + \pi \phi_n'(0)P \sum_{m=0}^{t-s-2} P^m \phi_n'(0)e \right\}
\]

= \pi \phi_n'(0) \left[ \sum_{s=0}^{t-1} \sum_{m=0}^{s-1} P^{s-m} \right] \phi_n'(0)e + \sum_{s=0}^{t-1} \pi \phi_n''(0)e + \pi \phi_n'(0) \left[ \sum_{s=0}^{t-1} \sum_{m=1}^{t-s-1} P^m \right] \phi_n'(0)e
\]

= t\pi \phi_n''(0)e + 2\pi \phi_n'(0) \left[ \sum_{s=0}^{t-1} \sum_{m=0}^{s-1} P^{s-m} \right] \phi_n'(0)e
\]

where the last equality follows from the fact that

\[
\sum_{s=0}^{t-1} \sum_{m=1}^{t-s-1} P^m = \sum_{s=0}^{t-1} \sum_{m=0}^{s-1} P^{s-m}.
\]

This completes the proof of Theorem 1.
Proof of Lemma 1

Since the wireless channel state process $s_n(t)$ of MS $n$ is time-reversible\(^2\), we can easily show that $S(t)$ is also time-reversible. Hence, there exist square matrices $V$ and $U$ and a diagonal matrix $\Lambda$ such that \[ P = V\Lambda U, \quad UV = I \]

where $I$ is the identity matrix and the components of $\Lambda$ are eigenvalues of $P$. It then follows that

\[
\frac{1}{t^2} \sum_{s=1}^{t-1} \sum_{m=0}^{s-1} P^{s-m} = \frac{1}{t^2} \sum_{s=1}^{t-1} (t-s)P^s \\
= \sum_{s=1}^{t-1} \frac{t-s}{t^2} V\Lambda^s U \\
= \sum_{s=1}^{t-1} \frac{t-s}{t^2} \left( \sum_{i=1}^{s} \lambda_i \right)U
\]

where $\lambda_i$ denotes the $i$-th eigenvalue of $P$ and $\Lambda = \text{diag}(\lambda_i)$. For convenience we let $\lambda_1 = 1$. Note that, since $P$ is a stochastic matrix, the eigenvalue $\lambda_1 = 1$ exists and by the Perron-Frobenius Theorem we know that $|\lambda_i| < 1$ for all $i = 2, \cdots, M$. In addition, since $P = Q_1 \otimes \cdots \otimes Q_N$, the first column of the matrix $V$ is $e$ and the first row of the matrix $U$ is $\pi$. Now observe that, for $|\lambda| < 1$,

\[
\sum_{s=1}^{t-1} \frac{t-s}{t^2} \lambda^s = \frac{1}{t} \sum_{s=1}^{t-1} \lambda^s - \frac{1}{t^2} \sum_{s=1}^{t-1} s\lambda^s \\
= \frac{1}{t} \frac{\lambda(1-\lambda^{t-1})}{1-\lambda} - \frac{1}{t^2} \left( \frac{\lambda(1-\lambda^{t-1})}{(1-\lambda)^2} + (t-1) \frac{\lambda^t}{1-\lambda} \right) \\
\to 0 \quad \text{as} \quad t \to \infty.
\]

\(^2s_n(t)\) is time-reversible if its transition probability matrix $Q_n$ satisfies $q_{n,0}(1-\beta_n) = q_{n,1}(1-\alpha_n)$ \cite{18}
For $\lambda = 1$,

$$\sum_{s=1}^{t-1} \frac{t - s}{t^2} \lambda^s = \sum_{s=1}^{t-1} \frac{t - s}{t^2} = \frac{1}{t^2} \left( t(t - 1) - \frac{(t - 1)t}{2} \right) \to \frac{1}{2} \text{ as } t \to \infty.$$ 

Hence, as $t \to \infty$, we get

$$\frac{1}{t^2} \sum_{s=1}^{t-1} \sum_{m=0}^{s-1} P^{s-m} \to V \left[ \text{diag} \left( \frac{1}{2}, 0, \ldots, 0 \right) \right] U = \frac{1}{2} e^{i \pi}.$$ 

This completes the proof of Lemma 1.

References


